

# NAVAL POSTGRADUATE SCHOOL Monterey, California



THESIS



An Investigation Into Backscattered Cross Section Calibration of an Acoustic Sounder Used for Analysis of Lower Atmospheric Turbulence

by

David Paul Davison, Jr.

December 1988

Thesis Advisor:

D. L. Walters

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An Investigation Into Backscattered Cross Section Calibration of an Acoustic Sounder Used for Analysis of Lower Atmospheric Turbulence

by

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Lieutenant, United States Navy
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Submitted in partial fulfillment of the requirements for the degree of

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# **ABSTRACT**

Atmospheric temperature structure fluctuations quantified by  ${\rm C_T}^2$  degrade the spatial and temporal coherence of electromagnetic and acoustic waves propagating in the atmosphere. A computer controlled atmospheric echosounder developed at the Naval Postgraduate School measures a time averaged  ${\rm C_T}^2$  profile of the lower atmosphere. Assigning the proper  ${\rm C_T}^2$  values to the backscattered return signals depends on an accurate calibration of this instrument. Calibration involves determination of the product of the echosounder's transmission efficiency  ${\rm E_t}$  and reception efficiency  ${\rm E_t}$ .

This thesis provides a preliminary investigation of a calibration process using pulsed acoustic energy backscattered from hard spheres. Supporting software calculates the desired product  $E_r E_t$  based on an assumption of echosounder efficiency reciprocity. Results of the calibration process investigation indicate this assumption may be invalid. The results also indicate the software performs as intended and that the proposed calibration method possesses sufficient merit to warrant further development.

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## I. INTRODUCTION

The variational effect of wind shears, convection, and temperature gradients on atmospheric density distributions characterize atmospheric turbulence. The temperature structure parameter,  $C_T^2$ , and the index of refraction structure parameter,  $C_n^2$ , quantify two density distributions that arise in acoustic and electromagnetic propagation. [Ref. 1]

The density distributions quantified by  $C_n^2$  and  $C_T^2$  induce phase fluctuations in propagating electromagnetic waves. The density distributions of atmospheric turbulence degrade the spatial and temporal coherences of initially coherent electromagnetic radiation. Similar, more pronounced effects occur in the propagation of acoustic waves. [Ref. 1]

A number of techniques for correcting these problems have been proposed, particularly with respect to electromagnetic propagation. These corrective techniques include the use of modular mirrors [Ref. 2], optical phase conjugation [Ref's. 2 and 3], optical and digital signal processing [Ref. 4], and simple avoidance of turbulent air masses. The appropriate timing of transmissions or suitable location of transmission and reception

facilities accomplishes avoidance [Ref's. 5 and 6]. These techniques can be applied alone or in combinations.

A knowledge of the history of the density or refractive structure along a proposed transmission path is required to effectively employ a number of these compensatory techniques. Time-averaged  $C_T^2$  or  $C_n^2$  profiles charactize this history [Ref's. 1 and 7].

Wroblewski and Weingartner developed and Moxcey refined a high resolution, computer controlled acoustic sounder, or echosounder, system. Their computer controlled echosounder system uses a transmitted pulse of acoustic energy as a probe of atmospheric structure. The interaction of the acoustic wave packet with atmospheric turbulence is used to develop a time-averaged  $C_T^2$  profile for a short spz\_ial range [Ref's. 5, 6 and 8]. The acoustic sounder may also be used to verify certain long range optical measurements of turbulence [Ref. 6].

Development of the  $C_T^2$  profile uses a  $C_T^2$  expression developed from the echosonde equation summarized by Neff [Ref. 7] and the empirical acoustic backscatter cross section per unit volume expression of Tatarski [Ref. 9] [Ref's. 5 and 6]. The accuracy of the absolute values of this  $C_T^2$  profile depends on the calibration of the acoustic antenna parameters

contained in the  ${\rm C_T}^2$  expression. The antenna parameters of interest are  ${\rm E_t}$ , the efficiency of conversion of electrical to acoustic power, and  ${\rm E_r}$ , the efficiency of conversion of acoustic power to electrical power.

In this thesis acoustic energy Lackscattered off acoustically hard spheres was used to determine  $\mathbf{E_r}$  and  $\mathbf{E_t}$ . Acoustic range equations were software implemented to perform the necessary calculations. Data and calculational results provided first order verification of the calibration process.

## II THEORETICAL BACKGROUND

# A. $C_T^2$ REVIEW

Wroblewski and Weingartner applied the echosonde equation,

$$P_{r} = E_{r} \left[ P_{t} E_{t} \right] \left[ e^{-2\alpha R} \right] \left[ \frac{C\tau}{2} \sigma_{0}(R,f) \right] \left[ \frac{A}{R^{2}} G \right]$$
 (1)

summarized by Neff [Ref. 7] and the acoustic backscattering cross section per unit volume expression given by Tatarski [Ref. 9] as

$$\sigma_0(R,f) = 0.0039 k^{1/3} \frac{C_T^2}{T_0^2}$$
 (2)

to develop the volume averaged expression

$$C_T^2 = \frac{1}{0.0039} \frac{1}{E_r E_r} \frac{T_0^2}{k^{1/3}} \frac{2}{CT} \frac{1}{AG} \frac{P_r}{P_r} R^2 e^{2\alpha R}$$
(3)

## where

- P<sub>r</sub> is the electrical power of the reflected signal returned to the acoustical antenna array,
- $E_r$  is the efficiency of conversion of the returned acoustic power to  $P_r$ ,
- P, is the electrical power supplied to the acoustical array,
- E, is the efficiency of conversion of P, to acoustic power,
- $\bullet$   $\alpha$  is the average attenuation per unit distance along the transmission path.
- R is the range from the array to the target or backscattering volume,

- c is the average speed of sound along the transmission path,
- τ is the length in time of the transmitted acoustic pulse,
- A is the antenna's aperture area,
- G is the antenna's effective aperture factor,
- k is the wavenumber of the incident acoustic energy, and
- T<sub>o</sub> is the average temperature (in degrees Kelvin) along the transmission path.

## [Ref's, 5 and 6]

The conditions at the time and location of each application determine the value of the terms  $T_{D}$ , k, c,  $\tau$ ,  $P_{r}$ ,  $P_{t}$ , R and  $\alpha$  in the  $C_{T}^{2}$  expression. The terms A and G depend on the antenna design.

Adverse field effects such as dust, debris and vibration loosened connections cause the remaining two terms,  $E_r$  and  $E_t$ , to change over the life of the system. This chapter develops the theoretical expressions supporting a determination of  $E_r$  and  $E_t$ .

## B. ACOUSTIC RANGE EQUATIONS AND EFFICIENCIES

In the following two sections the acoustic range equation of Neff [Ref. 7] is re-developed for targets of small area relative to the cross-section of the ensonifying field. Neff [Ref. 7], Skolnik [Ref. 10], and Probert-Jones [Ref. 11] were used for guidance.

# 1. One Way Range Equation and EtGo

As defined previously, the electrical power supplied to the acoustic array is  $P_t$ . It is converted at an efficiency  $E_t$  to an emitted acoustic power,

$$P_{\mathbf{a}} = P_{\mathbf{t}} E_{\mathbf{t}} . \tag{4}$$

 $\boldsymbol{P_{a}}$  is spread over a  $4\pi$  solid angle.

A far field range R is defined by the condition

$$\lambda/2 \gg |\mathbf{L}| - |\mathbf{R}|$$
 (5)

for the geometry in Figure 1.  $\lambda$  is the wavelength of the emitted acoustic wave.

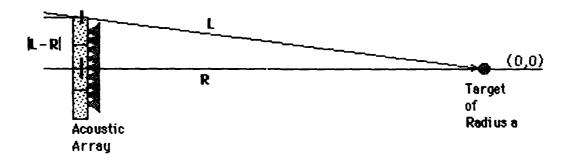


Figure 1: Calibration (Far Field) Geometry

The acoustic intensity,

$$I_a = P_a / 4\pi R^2, \tag{6}$$

describes the emitted acoustic power density of an isotropic emitter at a far field range R.  $I_a$  is in units of [W/m $^2$ ].

If the emitted power is directional, the intensity  $\mathbf{I}_{\mathbf{a}}$  is modulated by a geometrical gain,

$$G(\Omega) = G_0 |F(\Omega)|^2 = G_0 |F(\emptyset, \mathbf{e})|^2.$$
 (7)

Note this is not the same G found in equation (1). Now,

$$|F(0,0)|^2 = 1,$$
 (8)

and  $G_0$  is thefore the centerline gain of the acoustic array. [Ref. 11] The centerline direction (0,0) is taken to be normal to the face of the acoustic array. The acoustic intensity  $I_B$  is now written

$$I_8 = \frac{P_8}{4\pi R^2} G(\Omega). \tag{9}$$

In a real propagating medium intensity attenuation occurs along the transmission path. The radial term  $e^{-\alpha R}$  accounts for this attenuation when the atmospheric path is assumed to be reasonably

homogeneous. The acoustic intensity is then

$$I_{a} = \frac{P_{a}}{4\pi R^{2}} G(\Omega) e^{-\alpha R}.$$
 (10)

A target of cross sectional area  $\boldsymbol{A}_{tgt}$  intercepts a power

$$P_{tat} = I_8 A_{tat}. (11)$$

Expanding this expression with equation (10) yields

$$P_{tgt} = \left[\frac{P_{a}}{4\pi}\right] \left[e^{-\alpha R} G(\Omega)\right] \left[\frac{A_{tgt}}{R^2}\right].$$
 (12)

For a target of cross sectional area  $A_{tgt}$  with a sufficiently small solid angle  $A_{tgt}$  /R<sup>2</sup> little change in intensity occurs about the direction  $\Omega$ . Equation (12) is now arranged as a one way range equation along the centerline direction (0,0);

$$R = \left[ \frac{P_t E_t}{4\pi} e^{-\alpha R} G_0 \frac{A_{tgt}}{P_{tgt}} \right]^{\frac{1}{2}}.$$
 (13)

Isolating  $\mathbf{E_t}\mathbf{G_o}$  in the above expression yields

$$E_t G_0 = \frac{4\pi R^2}{P_t} e^{\alpha R} \frac{P_{tgt}}{A_{tat}}. \qquad (14)$$

Electrical power is expressed as

$$P = \frac{V_{rms}^2}{Z} = \frac{V_{ms}}{Z} \qquad (15)$$

where

- V<sub>rms</sub> is the empirical root mean square voltage,
- V<sub>ms</sub> is V<sub>rms</sub><sup>2</sup>, and
- Z is the electrical impedance.

 $E_{i}G_{0}$  is now written

$$E_t G_0 = 4\pi R^2 \frac{Z}{(V_{ms})_t} e^{\alpha R} \frac{P_{tot}}{A_{tot}} . \qquad (16)$$

The ratio  $P_{tgt}/A_{tgt}$  is defined as the acoustic intensity I received by the target. Therefore,

$$E_t G_0 = 4\pi R^2 \frac{Z}{(V_{ms})_t} e^{\alpha R} I_r$$
 (17)

# 2. Two Way Range Equation and Er

For an acoustically hard target the total power reflected is  $P_{tgt}$ .

The power reflected from the target surface has a normalized, directional intensity distribution described by the differential scattering cross section, **r**.

The direction from the target to the array is the backscattered direction. The differential scattering cross section in the backscattered direction is called the normalized backscattered cross section,  $\sigma_b$ .  $\sigma_b$  is defined as the power reflected toward the source per unit solid angle, normalized by the incident power density over a  $4\pi$  solid angle.

The backscattered intensity from the target is

$$I_{tgt} = \frac{P_{tgt}}{4\pi R^2} \sigma_b . \qquad (18)$$

An array of aperture area A at range R intercepts an attenuated, received acoustic power,

$$P_{ra} = I_{tat} A e^{-\alpha R}. (19)$$

Substituting equation (18) into equation (19) yields

$$P_{ra} = \frac{P_{tgt}}{4\pi R^2} \sigma_b e^{-\alpha R} A . \qquad (20)$$

 ${\rm P_{ra}}$  is converted to returned electrical power,  ${\rm P_{r}},$  at a return efficiency,  ${\rm E_{r}};$ 

$$P_r = P_{ra} E_r. (21)$$

Since electrical power is expressed as

$$P = \frac{V_{rms}^2}{Z} = \frac{V_{ms}}{Z} \qquad (15)$$

electrical power representing the returned acoustic signal from a target placed along an emitter's direction (0,0) is written as

$$\frac{(V_{ms})_r}{Z_r} = \frac{(V_{ms})_t}{Z_t} \left[ \frac{E_t G_0}{4\pi} e^{-\alpha R} \frac{A_{tgt}}{R^2} \right] \left[ \frac{E_r \sigma_b}{4\pi} e^{-\alpha R} \frac{A}{R^2} \right]. \tag{22}$$

For a passive, linear acoustic array circuit the concept of impedance reciprocity is invoked [Ref. 12] to yield a two way range equation for scattering from a small target;

$$R = \left[\frac{(V_{ms})_t}{(V_{ms})_r} E_t E_r \frac{G_0 \sigma_b}{(4\pi)^2} e^{-2\alpha R} A A_{tgt}\right]^{\frac{1}{4}}.$$
 (23)

This is written to isolate Er;

$$E_{r} = \frac{(V_{ms})_{r}}{(V_{ms})_{t}} \frac{(4\pi R^{2})^{2}}{E_{t}G_{0}} \frac{e^{2\alpha R}}{A A_{tat}} \frac{1}{\sigma_{b}} . \qquad (24)$$

A part of the return signal's power is unwanted noise.

Subtracting noise power from the returned acoustic power yields

$$E_{r} = \frac{(V_{ms})_{r} - (V_{ms})_{n}}{(V_{ms})_{t}} \frac{(4\pi R^{2})^{2}}{E_{t}G_{0}} \frac{e^{2\alpha R}}{A A_{tot}} \frac{1}{\sigma_{b}} . (25)$$

As return voltages are quite small they are analyzed after electrical amplification. Calling the electrical amplification a gain  $G_a$  allows introduction of the final modification to  $E_r$ :

$$E_{r} = \frac{(V_{ms})_{r} - (V_{ms})_{h}}{(V_{ms})_{t} G_{e}^{2}} \frac{(4\pi R^{2})^{2}}{E_{t} G_{0}} \frac{e^{2\alpha R}}{A A_{tgt}} \frac{1}{\sigma_{b}} . (26)$$

## C. ATTENUATION AND RANGE DETERMINATION

Employing equations (17) and (26) in the calibration process requires calculating the attenuation  $\alpha$  and the range R from the array to the target.

#### 1. Attenuation

The calculated attenuation  $\alpha$  for an atmospheric propagation path is assumed to be the combined result of molecular and classical absorption in the atmosphere. The classical absorption  $\alpha_{cl}$  is attributed to heat conduction and viscous effects while the larger molecular absorption  $\alpha_{mol}$  is attributed to the excitation of internal energy modes of atmospheric gases by the propagating sound energy. [Ref. 7]

Information from Neff [Ref. 7], Businger [Ref. 13], Neiburger [Ref. 14], and Fuller [Ref. 15] is applied to make a reasonable determination of  $\alpha$ . The determination is made using the average measured temperature

 ${\rm T_c}$  in degrees Celsius along the propagation path, the measured atmospheric pressure P in millibars, and the measured relative humidity Rh in percent.

The molecular attenuation coefficient,  $\alpha_{\text{mol}}$ , is expressed by the empirical relationship

$$\alpha_{\text{mol}} = \frac{\alpha_{\text{max}}}{304.8} \left[ \left( 0.18 \, f_{\text{ratio}} \right)^2 + \left( \frac{2 \left( f_{\text{ratio}} \right)^2}{1 + f_{\text{ratio}}^2} \right)^2 \right]^{\frac{1}{2}}.$$
 (27)

 $\alpha_{mol}$  is in units of [-dB/m].

Now,

$$\alpha_{\text{max}} = 0.0078 \text{ f}_{\text{m}} (T*)^{-2.5} e^{7.77(1-1/T*)}$$
 (28)

is the maximum absorption at

$$f_{\rm m} = \frac{(10 + 6600 (100 \frac{e}{P}) + 44,400 (100 \frac{e}{P})^2) P^*}{(T^*)^{0.8}}$$
(29)

 $f_m$  is in units of [Hz].

Also.

$$\frac{e}{P} = \frac{E_s Rh}{P - E_s (1 + Rh)} \tag{30}$$

is the mole ratio of water vapor in the atmosphere. The water vapor pressure e is expressed in millibars.

$$E_{s} = 10^{(9.4 - 2353/T_{0})}$$
 (31)

expresses the saturation vapor pressure  $\mathbf{E_{s}}$ .  $\mathbf{T_{o}}$  is the average absolute temperature along the transmission path, as defined in Section A.

$$T* = (1.8 T_c + 492) / 519,$$
 (32)

and

$$P* = P / 1014$$
. (33)

Finally,

$$f_{\text{retio}} = f / f_{\text{m}},$$
 (34)

where f is the operating frequency of the echosounder in [Hz]. [Ref's. 7, 13, 14 and 15]

The classical attenuation coefficient,  $\alpha_{\text{cl}}$ , is approximated by

$$\alpha_{cl} = 1.74 \times 10^{-10} (f)^2$$
 (35)

 $\alpha_{cl}$  is in units of [-dB/m]. [Ref. 7]

For a total attenuation

$$\alpha_{T} = \alpha_{c1} + \alpha_{mo1} \tag{36}$$

and the relationship

$$-\alpha_{\tau}R = 10 \log e^{-\alpha R}$$
 (37)

the average attenuation per meter is finally written as

$$\alpha = \frac{\alpha_{cl} + \alpha_{mol}}{10} \ln 10$$
 (38)

Equation (37) is derived from unit definitions and equation one of Neff [Ref. 7].

# 2. Range

The coordinate system for the backscattered cross section determination uses the center of the target spheres as the coordinate system origin. Therefore, the radius a of a target sphere and the time of return  $\mathbf{t_r}$  of an acoustic signal from the acoustic echosounder are used to determine the range R from the target to the echosounder;

$$R = \frac{ct_r}{2} + a . ag{39}$$

c is the average speed of sound along the transmission path, and

$$t_r = t_{receive} - t_{transmit}. (40)$$

For the range of temperatures expected in normal echosounder operation the speed of sound in meter per seconds in dry air is

$$c_{dry} = 20.05 \sqrt{T_0}$$
 (41)

For moist air c becomes

$$c = c_{moist} = c_{dry} (1 + 0.14e/P),$$
 (42)

where e/P is defined in equation (30). [Ref. 1]

## D. BACKSCATTERED CROSS SECTION DETERMINATION

Employing equations (17) and (26) in the calibration process also requires calculating  $\sigma_b$ , the backscattered cross section. Recall from Paragraph B.2 that the backscattered cross section is the differential scattering cross section in the direction of the energy source.

## 1. Initial Conditions and Assumptions

In determining the differential scattering cross section  ${\bf r}$  it is assumed the incident wave state is unchanged after scattering. That is, for a stationary, acoustically hard target it is assumed

$$k = k_{incident} = k_{scattered}$$
, (43)

and that incident k has good definition, i.e. the incident wave can be considered monochromatic [Ref. 16].

Selection of acoustically hard spheres as targets further simplifies the determination of the differential scattering cross section solution. This target selection sets the boundary condition at the target

surface as the Neumann boundary condition

$$\frac{\partial V}{\partial n} = 0. (44)$$

 $oldsymbol{\mathcal{V}}$  is the velocity potential defined by the acoustic propagation velocity c as

$$C = \nabla V. \tag{45}$$

n lies along the direction normal to the target surface. Also,

$$p = -p_0 \frac{\partial V}{\partial t} \tag{46}$$

for the pressure p and rest density  $\rho_o$  of the propagating medium. [Ref. 16].

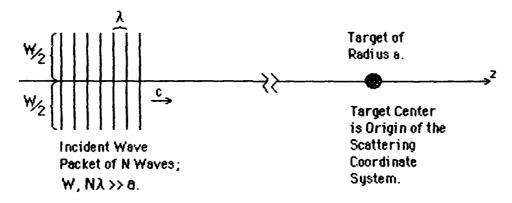


Figure 2: Incident Wave and Target Description

The target sphere is aligned along the centroid of the main acoustic lobe of the acoustic arrray echosounder, that is, along the direction (0,0), normal to the face of the array. The target sphere is

selected to be sufficiently small with respect to incident beam dimensions so that an essentially uniform intensity is intercepted by the sphere [Ref. 17]. Since the target intercepts a very small proportion of the centroid of the transmitted sounding lobe no accommodation for sounding lobe geometry over the target's angular area is made. Acoustic gain is simply taken as  $G_0$ .

Target placement is also at a far field range. This allows spherical incident waves to be treated as planar incident waves and places the target outside the range associated with the ring time of the echosounder [Ref. 8]. Placement of the target is also close enough to the array to support an assumption of transmission medium homogeneity along the transmission path.

Finally, the potential describing the target falls off faster than 1/r. The describing potential for an acoustically hard sphere with radius a is

$$V(\mathbf{r}) = \begin{cases} \infty & \Gamma \le \mathbf{a} \\ 0 & \Gamma \ge \mathbf{a} \end{cases} \tag{47}$$

Thus, for  $r > a \ V(r)$  falls off faster than 1/r.

Note that with the exception of the specific surface boundary condition for acoustic waves the above conditions and assumptions closely correspond to those for plane electromagnetic waves incident on a perfectly conducting sphere, and for perfectly elastic nuclear scattering in a center of mass coordinate system. [Ref's. 16, 18, 19 and 20]

## 2. The Incident Wave

The asymptotic form of an incident plane wave approaching a scattering target but beyond the influence of the target's potential  $V(\mathbf{r})$  is the undisturbed, or free, plane wave. In the spherical polar coordinates (r,ø,ø) illustrated in Figure 3 a free, monochromatic plane wave of unit amplitude propagating in the direction  $\Omega_o$  given by  $(\mathfrak{s}_o,\mathfrak{s}_o)$  is

$$f(\mathbf{r},t) = e^{ik\mathbf{r}[\cos(\theta_0)\cos(\theta) + \sin(\theta_0)\cos(\theta - \theta_0)]} e^{-i\omega t}. (48)$$

[Ref. 16]

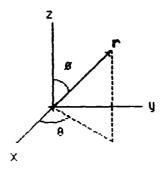


Figure 3: Spherical Polar Coordinates

For a wave of unit amplitude incident along the z axis ,  $\Omega_{o}$  = 0 and the free wave  $f(\mathbf{r},t)$  is presented as

$$\int (\mathbf{r}, t) = e^{ik\mathbf{r}\cos(\emptyset)} e^{-i\omega t} = e^{ik\mathbf{z}} e^{-i\omega t}. \tag{49}$$

Note this is a solution with axial symmetry for the free wave equation [Ref. 18].

The wave equation describing the behavior of the undisturbed or free monochromatic wave  $\{(\mathbf{r},t)\}$  is

$$(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) f(\mathbf{r}, t) = 0$$
 (50)

c is the speed of propagation of the free wave. If the wave function  $f(\mathbf{r},t)$  has a simple harmonic time dependence, i.e.,

$$f(\mathbf{r},t) = e^{-i\omega t} f(\mathbf{r}), \qquad (51)$$

then for a wave number

$$k = \omega/c = 2\pi/\lambda \tag{52}$$

the free wave equation becomes the Helmholtz equation

$$[\nabla^2 + k^2] f(r,t) = 0$$
 (53)

[Ref. 21].

The stationary solution  $f(\mathbf{r})$  of the free wave equation can be written as the product

$$f(r) = f_1(q_1) f_2(q_2) f_3(q_3)$$
 (54)

for orthogonal curvilinear coordinates  $q_1$ ,  $q_2$ ,  $q_3$  [Ref. 16]. Recall an undisturbed plane wave has a cylindrical symmetry about the axis of propagation [Ref. 22]. Using the spherical polar coordinates

$$q_1 = r, q_2 = \emptyset, q_3 = \emptyset,$$
 (55)

the stationary solution describing an undisturbed plane wave propagating along the z axis is

$$f(\mathbf{r}) = f_1(\mathbf{r}) f_2(\emptyset)$$
 (56)

Substituting  $f(\mathbf{r})$  into the free wave equation expressed in spherical coordinates and separating the coordinate dependencies yields the separated free wave equation,

$$(kr)^2 + \frac{1}{f_1} \frac{d}{dr} r^2 \frac{d}{dr} f_1 = -\frac{1}{f_2 \sin g} \frac{d}{dg} (\sin g \frac{d}{dg} f_2).$$
 (57)

Using a separation constant of 1(1+1) the equation for the angular dependency of the free wave becomes

$$\frac{1}{\int_2 \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d}{d\theta} \int_2 1 + 1(1+1) = 0 \right) . (58)$$

Letting

$$\cos(\emptyset) = \chi \tag{59}$$

in the angular equation yields

$$[(1-x^2)\frac{d^2}{dx^2}-2x\frac{d}{dx}+1(1+1)]f_2=0. (60)$$

The previous equation is known as the Legendre differential equation [Ref. 21]. The most useful solutions to the Legendre differential equation are the Legendre polynomials, designated by  $P_{\parallel}(x)$ . They can be generated by the recursion relationships

$$P_{n}(x) = 1, (61a)$$

$$P_1(x) = x \tag{61b}$$

and

$$P_1(x) = [2-1/1] \times P_{1-1}(x) - [1-1/1] P_{1-2}(x)$$
 (61c)

[Ref. 21]. Therefore,

$$\int_{2,1}(\emptyset) = P_1(\cos \emptyset) . \tag{62}$$

From equation (57), the equation describing the radial dependency of the free wave is

$$\left\{\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \left[k^2 - \frac{1(1+1)}{r^2}\right]\right\} f_1 = 0$$
 (63)

Changing scale to the variable s = kr yields the differential equation

$$\left\{\frac{d^2}{ds^2} + \frac{2}{s}\frac{d}{ds} + \left[1 - \frac{1(1+1)}{s^2}\right]\right\}f_1 = 0$$
 (64)

[Ref. 17].

This differential equation has the spherical Bessel function  $j_l(s)$  with amplitude  $A_l$  as a solution;

$$\int_{1,1} (s) = A_1 j_1(s)$$
 (65)

[Ref. 18].

The stationary solution of the free wave equation in spherical coordinates is now written as the partial wave

$$j_1(r) = P_1(\cos \emptyset) [A, j_1(kr)].$$
 (66)

The most general form of this free wave equation solution is a Fourier-Bessel series,

$$\int (r, \emptyset) = \sum_{i=0}^{\infty} A_i j_i(kr) P_i(\cos \emptyset)$$
 (67)

[Ref's. 17 and 21].

A plane, free wave of unit amplitude propagating in the direction  $\Omega_0 = 0 \text{ is then}$ 

$$e^{ikz} = \sum_{l=0}^{\infty} A_l j_l(kr) P_l(\cos \emptyset) . \qquad (68)$$

The norm of the Legendre polynomials, the asymptotic expression

$$j_1(kr) \sim \frac{\sin(kr - \frac{\pi}{2}1)}{kr}$$
 (69)

and the process detailed in Elton [Ref. 18] are used to determine A,;

$$A_1 = i^1 [21 + 1] . (70)$$

Therefore,

$$e^{ikz} = \sum_{l=0}^{\infty} i^{l} [2l+1] j_{l}(kr) P_{l}(\cos \theta)$$
 (71)

[Ref's. 18 and 21].

## 3. Detection Conditions and Assumptions

As the dimensions of the incident acoustic wave packet are greater than the size of the target it is expected that a portion of the incident wave packet continues to propagate past the target. The total or disturbed wave description is then a superposition of the incident and scattered wave descriptions. [Ref. 17]

The acoustic array serves as both transmitter and receiver, or detector. This keeps the detector outside the path of continued propagation of the incident wave packet. [Ref. 17] Therefore, only the scattered wave description is of interest in the calibration of the array.

The placement of the target at a far field range simultaneously places the detector in the region of asymptotic behavior of the scattered wave.

#### 4. The Disturbed Wave

Once an incident wave described by the velocity potential  $V(\mathbf{r})$  enters the region of influence of a scattering target's spherical potential  $V(\mathbf{r})$  the wave equation becomes the disturbed wave equation

$$\{ \nabla^2 + [k^2 - V(r)] \} V(r) = 0$$
 (72)

[Ref. 17]. Comparison of the form of the disturbed wave equation to the free wave equation gives an expectation of a solution of the form

$$V(\mathbf{r}) = V_1(\mathbf{r}) V_2(\mathbf{g}) . \tag{73}$$

Substituting  $V(\mathbf{r})$  into the disturbed wave equation and separating variables yields

$$-\frac{1}{V_2 \sin \theta} \frac{d}{d\theta} (\sin \theta \frac{d}{d\theta}) V_2 = r^2 [k^2 - V(r)] + \frac{1}{V_1} \frac{d}{dr} r^2 \frac{d}{dr} V_1 . (74)$$

The angular equation for the disturbed wave is identical in form to the angular equation for the free wave and independent of the Neumann boundary condition  $\partial \mathcal{V} \partial r = 0$ . Therefore,

$$V_{2,l}(\emptyset) = P_l(\cos \emptyset). \tag{75}$$

Using the previous substitution of s = kr yields the now modified

Bessel equation as the radial equation for the disturbed wave;

$$\left\{\frac{d^2}{ds^2} + \frac{2}{s}\frac{d}{ds} + \left[1 - \frac{1(1+1) + V(r)}{s^2}\right]\right\}V_1 = 0. \quad (76)$$

Following the solution development in Elton [Ref. 18] and applying the Neumann boundary condition at r = a leads to a solution

$$V_{1,l}(r) = i^{l} [2l + 1] [j_{l}(kr) - a_{l}' h_{l}^{(1)}(kr)]$$
 (77)

The relationship

$$a_1' = i e^{-i\delta_1} \sin \delta_1 = \frac{j_1'(ka)}{h_1^{(1)'}(ka)}$$
 (78)

is defined from Morse [Ref. 17] and Bowman [Ref. 16]. Additionally,

- the j<sub>1</sub>(kr) are the spherical Bessel functions of the first kind, argument kr, and order 1,
- h<sub>i</sub>(1)(kr) are the Hankel functions of argument kr and order 1,
- the δ<sub>1</sub> are the phase shifts of the usual partial wave description for spherical scattering solutions [Ref's. 17 and 20],
- the j<sub>i</sub>'(ka) are the first derivatives of the j<sub>i</sub>(kr), taken with respect
  to the argument kr and evaluated at r = a,

• 
$$h_i^{(1)}(ka) = j_i'(ka) + i y_i'(ka)$$
, and (79)

• the  $y_l$  (ka) are the first derivatives of the  $y_l$  (kr), the spherical Bessel functions of the second kind with argument kr and order 1, taken with respect to the argument kr and evaluated at r = a.

The general series solution for the total disturbed wave  $V(\mathbf{r})$  is then

$$V(\mathbf{r}) = \sum_{l=0}^{\infty} i^{l} [2l+1] P_{l}(\cos \theta) [j_{l}(kr) - a_{l}' h_{l}^{(1)}(kr)] . (80)$$

# 5. The Scattered Wave and Its Amplitude

The general asymptotic form for a disturbed wave  $V(r,\Omega)$  resulting from a plane wave  $V_i(r,\Omega)$  incident along  $\Omega_0$  = 0 and scattered by a spherical potential V(r) is

$$V(\Gamma,\Omega) \cong e^{ikz} + s(\Omega) e^{ikr}/\Gamma$$
 (81)

[Ref. 18]. This description of the asymptotic disturbed wave is a superposition of the incident plane wave  $e^{ikz}$  and a spherical, scattered wave  $s(\Omega)e^{ikr}/r$ . The term  $s(\Omega)$  is called the scattering amplitude [Ref's. 18, 19 and 22].

Now, recalling the series equivalence for the plane wave incident along  $\Omega_0$  in equation (71), and comparing the general series solution for the disturbed wave in equation (80) to its asymptotic form in equation (81) gives

$$s(\Omega)e^{ikr}/r = -\sum_{l=0}^{\infty} i^{l}[2l+1] P_{l}(\cos \emptyset) a_{l}^{t} h_{l}^{(1)}(kr)$$
. (82)

This is a series expression for the scattered wave. Therefore,

$$s(\Omega) = \frac{\sqrt{4\pi}}{k} i \sum_{l=0}^{\infty} [2l+1] P_{l}(\cos \emptyset) a_{l}$$
 (83)

[Ref's. 16 and 18].

#### 6. The Backscattered Cross Section

The scattering amplitude s( $\Omega$ ) is related to the differential scattering cross section  $\sigma(\Omega)$  =  $\sigma A_{tqt}$  by

$$\sigma(\Omega) = |s(\Omega)|^2 \tag{84}$$

[Ref's. 18, 19 and 22].

Applying the series expression for the scattering amplitude in equation (83) to the previous equation yields

$$\sigma A_{tgt} = \sigma(\Omega) = \frac{4\pi}{L^2} |\sum_{l=0}^{\infty} i[2l+1] a_l' P_l(\cos \theta)|^2$$
. (85)

For the backscattered direction  $\phi = \pi$  this gives a backscattered cross section of

$$\sigma_b A_{tgt} = \sigma_B = \frac{4\pi}{k^2} |\sum_{1=0}^{\infty} [21+1] a_1'|^2$$
 (86a)

[Ref's. 16 and 17]. For the spherical target  $A_{tqt} = \pi a^2$ , so

$$\sigma_{b} = \frac{4}{(ka)^{2}} \sum_{l=0}^{\infty} [2l+1] a_{l}^{l} |^{2}.$$
 (86b)

It should be noted a plane wave of unit amplitude incident from the positive z direction and backscattered along  $\emptyset = 0$  would introduce a factor of  $(-1)^I$  into the previous sum [Ref. 16].

#### E. SPHERICAL BESSEL FUNCTIONS AND THEIR DERIVATIVES

Implementation of the previous equation necessitates calculation of the  $\mathbf{a_i}^{\star}$ . The recursion relations

$$j_0'(ka) = [-j_0(ka) + \cos(ka)]/ka$$
, (87a)

$$j_1'(ka) = \{-[1+1][j_1(ka)]/ka\} + j_{1-1}(ka),$$
 (87b)

$$y_n'(ka) = [-y_n(ka) + \sin(ka)]/ka$$
 (87c)

and

$$y_1'(ka) = \{-[1+1][y_1(ka)]/ka\} + y_{1-1}(ka)$$
 (87d)

determine the  $j_i$  (ka) and  $y_i$  (ka) needed for the calculation of  $a_i$  in equation (78) [Ref's. 21 and 23].

Determination of the regular and irregular spherical Bessel functions at r = a supports the use of the recursion relations in equations (87). As ka is real the irregular spherical Bessel functions are stable enough to use the recursion relations

$$y_0(ka) = -[cos(ka)]/ka$$
, (88a)

$$y_1(ka) = [y_0(ka) - \sin(ka)]/ka$$
 (88b)

and

$$y_1(ka) = \{[21 - 1][y_{1-1}(ka)]/ka\} - y_{1-2}(ka)$$
 (88c)

in their determination [Ref's. 23 and 24].

Because the size of the argument of the regular spherical Bessel functions may be very small the method of negative continued fractions is appropriate to the fast and accurate determination of a converging solution to these functions [Ref.24]. The regular spherical Bessel function reciprocals are therefore determined by the negative continued fraction calculation

$$\frac{1}{j_{1}(ka)} = \frac{1}{j_{0}(ka)} \frac{[-][b_{1,1}][b_{1,2},b_{1,1}]...[b_{1,1}][b_{1,2},b_{1,1}]...}{[b_{1,2}].....[b_{1,2}][b_{1,3},b_{1,2}]...} (89)$$

Now,

$$b_{1,m} = 2(1 + m - 1)/ka$$
 (90)

and

$$[-][b_{1,m}, \dots, b_{1,2}, b_{1,1}] = b_{1,m} - \frac{1}{b_{1,m-1}} - \dots - \frac{1}{b_{1,2}} - \frac{1}{b_{1,1}}$$

$$= b_{1,m} - \frac{1}{b_{1,m-1} - \frac{1}{b_{1,m-2} - \frac{1}{b_{1,m-3}}}$$

$$= b_{1,m-2} - \frac{1}{b_{1,m-3} - \dots - \frac{1}{b_{1,2} - \frac{1}{b_{1,1}}}}$$

$$(91)$$

[Ref. 25].

While the regular spherical Bessel function reciprocal calculation of equation (89) would appear to involve an infinite number of terms, the calculation can in fact be terminated when the m<sup>th</sup> numerator term equals the m<sup>th</sup> denominator term to the desired precision for the spherical Bessel function's solution. [Ref's. 24 and 25]

## III. CALIBRATION PROCESS AND SOFTWARE

Small quantities of data were collected in the anechoic chamber at the Naval Postgraduate School. This data aided first order verification of the calibration process and its supporting software. A brief description of the anechoic chamber is provided in Appendix A.

The calibration process and data development occured in two stages. The first stage used equation (26) and two way propagation path measurements to determine  $\mathbf{e}_b \mathbf{E}_r \mathbf{E}_t \mathbf{G}_0$ . The second stage determined  $\mathbf{E}_t \mathbf{G}_0$  using equation (17) and one way propagation path measurements.

 ${\rm E_r}$  was then determined from  ${\bf \sigma_b}{\rm E_r}{\rm E_t}{\rm G_0},~{\rm E_t}{\rm G_0}$  and the calculated  ${\bf \sigma_b}$  of equation (86b).

Because reciprocity was assumed for  $\mathbf{E_r}$  and  $\mathbf{E_t}$ 

$$E_r E_t = E_r^2 \tag{92}$$

provided the desired calibration information.

Supporting software was written to perform calculations required for calibration. The next section describes the development of this software.

#### A. CALIBRATION SOFTWARE DEVELOPMENT

Calibration software was written in BASIC. It was used with a BASIC 4.0 compiler on a Hewlett Packard 310 computer. Computer performance was enhanced by an Infotek floating point processor.

Notation in the programs sometimes differs from that in the thesis text as a result of H.P. keyboard limitations. Variable definitions are located at the beginning of subprograms or modules. These definitions are provided throughout a program as an aid in determining program to text variable equivalence.

The main effort of software development was calculation of backscattered cross sections. This was done with equation (86b).

Information available for summation loop verification was differential scattering cross section information. Differential scattering section software was developed first.

1. Differential Scattering Cross Section Software Development

The differential scattering cross sections • were calculated using equation (85) and equation (78).

To support equation (78) employment the subprograms calculating the values of the Legendre polynomials and the values of the

regular and irregular spherical Bessel functions were developed first.

Each subprogram was developed and tested separately in test programs.

Each test program was designed to provide a tabular output for verification against values found in Abramowitz and Stegun [Ref. 23].

The Legendre values were calculated using equations (61). The irregular spherical Bessel functions were calculated using equations (88). The regular spherical Bessel functions were calculated using equations (89), (90) and (91).

Following verification these subprograms were assembled with subprograms calculating the derivatives of the regular and irregular spherical Bessel functions, the normalized differential scattering cross section  $\sigma$ , the scattering modulus of Bowman [Ref. 16], and with three graphics subprograms. This assembly created the differential scattering cross section program found in Appendix B.

The values of the regular and irregular spherical Bessel function derivatives at r=a were calculated using equations (87).

Differential scattering cross sections were calculated using equation (85). The number of orders I required for the summation calculation in equation (85) was determined by generating tabular outputs

and determining the number of orders required for numerical stability to occur to 12 decimal places. This stability process was based on the concept of the phase shift  $\delta_1$  of each outgoing partial scattered wave becoming progressively smaller with increasing I until  $a_1$  (equation (78)) became negligible for the precision desired [Ref's. 18, 19, 20 and 22].

Tabular outputs covered a ka value range of one to thirty at varying scattering angles, including  $\emptyset = \pi$ . This ka value range selection was based on the expected maximum target size to be encountered during actual echosounder employment.

Twenty-one orders greater than the integer value of the argument ka were determined to be required for equation (85)'s summation. The number of orders required was greater than the usually cited relationship

$$ka \approx \sqrt{1(1+1)} \tag{93}$$

gives. It should be noted that the precision criteria of **a** was greater than that normally encountered. [Ref's. 18, 19, 20 and 22]

The graphics subprograms of the differential scattering cross section program provided outputs for comparisons to Figure 80 in

Morse [Ref. 17] and Figure 10.10 of Bowman [Ref. 16]. These comparisons were used to verify the functioning and output of the differential scattering cross section program.

The graphics subprograms used to generate Figure 4 provide linear polar plots of the differential scattering cross section  $\sigma(\Omega)$  normalized by the cross sectional area  $A_{tgt}$  of the target sphere. These linear polar plots show the normalized differential scattering cross section of acoustically hard spheres ensonified by plane acoustic waves incident from the left, i.e. from  $\emptyset=\pi$ .

The first view of each sheet in Figure 4 shows the entire normalized differential scattering cross section pattern, including the forward lobe, or "shadow zone", along  $\theta = 0$ . The second view of each sheet is a magnified plot showing the scattering detail at a scale comparable to the normalized backscattered cross section value. Note the two views are identical on Sheet 1. The need for magnification with increasing ka becomes apparent on subsequent sheets.

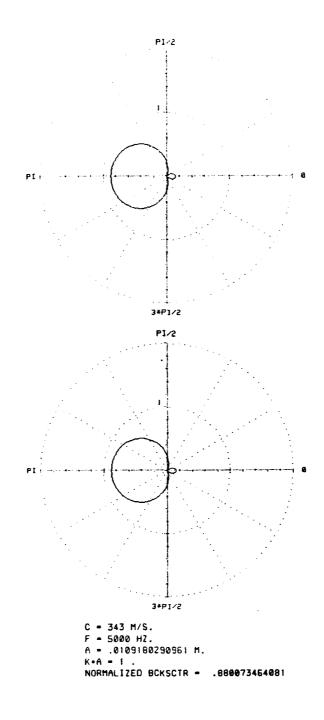


Figure 4, Sheet 1 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].

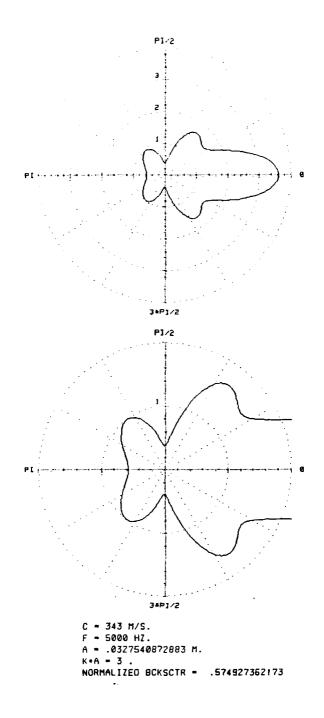


Figure 4, Sheet 2 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].

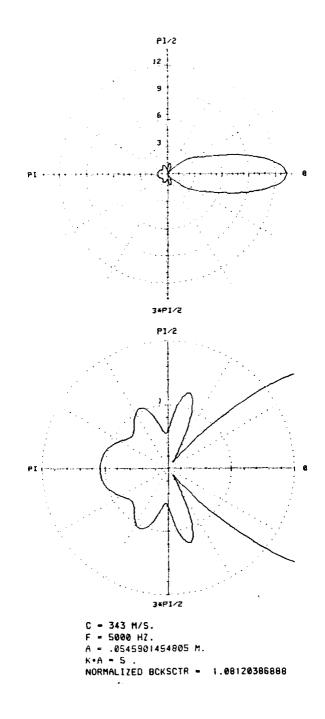


Figure 4, Sheet 3 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].

Figure 4 was used for comparison to Figure 80 of Morse [Ref. 17].

The radial scale of Morse was not graduated so verification consisted of a check for the correct number and approximate angular location of minimums and maximums in the differential scattering cross section patterns.

The graphics subprograms used to generate Figure 5 provide semi-logarithmic plots of the scattering modulus *S* of Bowman [Ref. 16].

$$5 = k s(\emptyset) / \sqrt{4\pi}$$
 (94)

is the scattering modulus S of Bowman [Ref. 16].  $s(\emptyset)$  is the scattering amplitude  $s(\Omega)$ , as defined by equation (83), with a factor of  $(-1)^l$  introduced into the sum, that is,

$$s(\emptyset) = \frac{\sqrt{4\pi}}{k} i \sum_{l=0}^{\infty} (-1)^{l} (2l+1) P_{l}(\cos \emptyset) a_{l}^{l}. \quad (95)$$

 $s(\Omega)$  is normally used to calculate  $\sigma(\Omega)$ , per equation (84). The scattering modulus S of Bowman [Ref. 16] could also be used to calculate  $\sigma(\Omega)$ . Note the introduction of  $(-1)^l$  into the summation corresponds to the case of an acoustically hard sphere ensonified by a plane acoustic wave incident from  $\emptyset = 0$ .

Comparison of Figure 5 to Figure 10.10 of Bowman [Ref. 16] shows a match in both  $\emptyset$  and S. This graphical match verifies the functioning and output of the differential scattering cross section program of Appendix B.

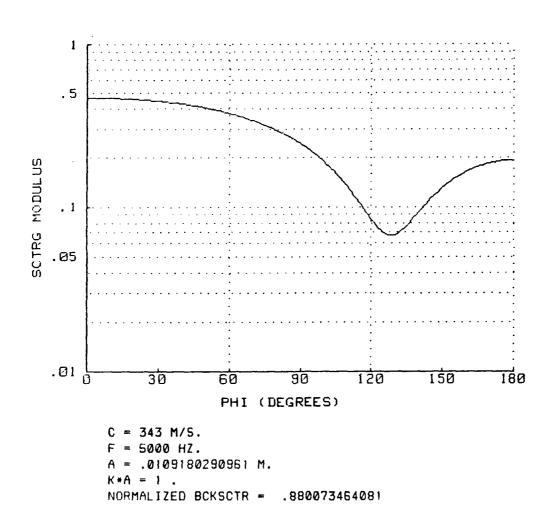
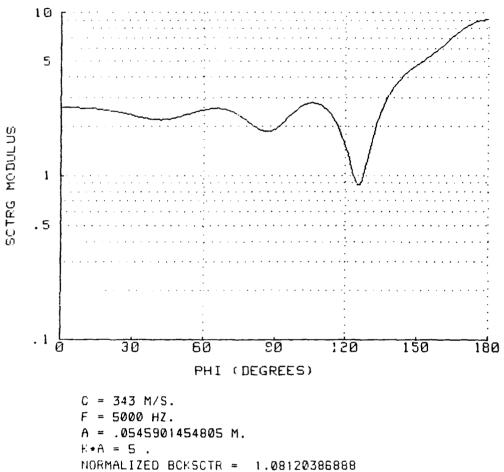


Figure 5, Sheet 1 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].



NORMALIZED BCKSCTR = 1.08120386888

Figure 5, Sheet 2 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].

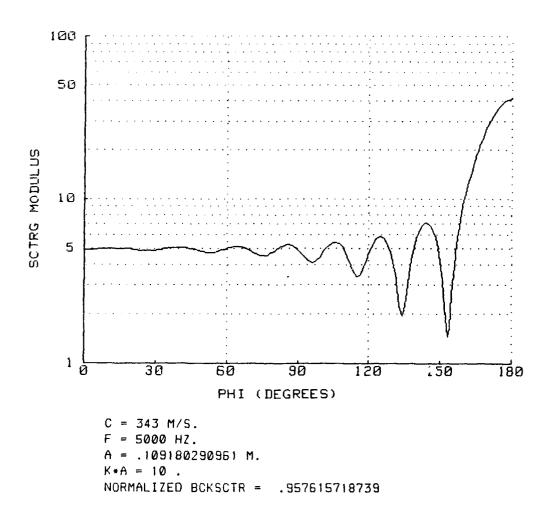


Figure 5, Sheet 3 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].

# 2. Backscattered Cross Section Software Development

The differential scattering cross section program of Appendix B was modified to yield the backscattered cross section program of Appendix C. With the modification the normalized differential scattering cross section is calculated for the backscattered direction only. Equation (86b) is used for this calculation.

Additionally, the graphics subprograms of the differential scattering cross section program were replaced by one graphics subprogram. The new graphics subprogram was designed to provide a rectangular graph of the normalized backscattered cross section,  $\sigma_b$ .  $\sigma_b$  is plotted as a function of ka for a specifiable range of values of ka.

Figure 6 was generated by the backscattered cross section program. It was used for comparison to Figure 10.11 of Bowman [Ref. 16]. This comparison served as a verification of the calculated, normalized backscattered cross section values. Note the ka axis is labeled K\*A.

Figure 7 was generated to verify the normalized backscattered cross section profile was approaching a value of one as ka left the Mie scattering regime and entered the "geometric optics" region. Note the ka axis is labeled K\*A.

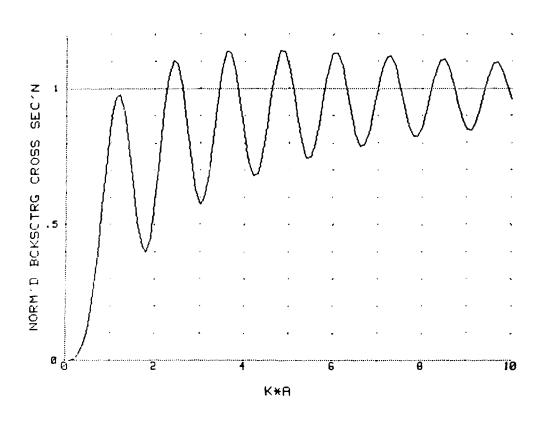


Figure 6: Backscattered Cross Section Program Output for Verification of Calculated Normalized Backscattered Cross Section Values by Comparison to Figure 10.11 of Bowman [Ref. 16].

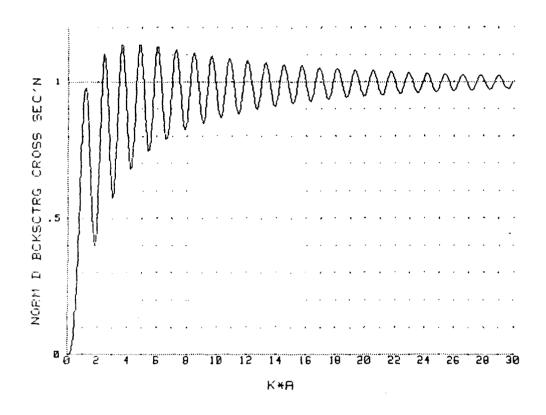


Figure 7: Backscattered Cross Section Program Output for Verification of the Normalized Backscattered Cross Section Asymptotic Behavior with Increasing ka.

## 3. Acoustic Array Efficiency Software Development

The acoustic array efficiency program in Appendix D determined  $E_r \, E_t$ . In general, each major section of Chapter II of this thesis and each set of analyzed data is represented by a subprogram of the acoustic array efficiency program.

The acoustic array efficiency program of Appendix D includes the non-graphics subprograms present in the backscattered cross section program of Appendix C. It also includes subprograms designed to furnish stored data, calculate attenuation and range using equations (27) through (42), and use developed information to calculate  $E_tG_0$  with equation (17).

Equation (86b) was used to calculate  $\sigma_h$ .

The appropriate subprogram and main program calculational results were then used in equation (26) to determine  $E_r$ .  $E_r$  was used in equation (92) to determine  $E_r$   $E_t$ .

#### B. DATA ACQUISITION AND SUPPORTING EQUIPMENT

Data collection occured 28 November 1987, 30 November 1987, and 7 December 1987.

Data collected 28 November and 30 November provided information needed for calculation of  $\mathbf{e}_b \mathbf{E}_r \mathbf{E}_t \mathbf{G}_0$ . Data collected 7 December allowed calculation of  $\mathbf{E}_t \mathbf{G}_0$  and the calculation of  $\mathbf{E}_r$ 's.

Collected data is presented in Appendix E.

## 1. Two Way Propagation Path Measurements

Two way propagation path data were collected 28 and 30 November. Measurements were used to calculate the product  $\sigma_b E_r E_t G_0$ .

Figure 8 provides a connection schematic for the data acquisiton equipment.

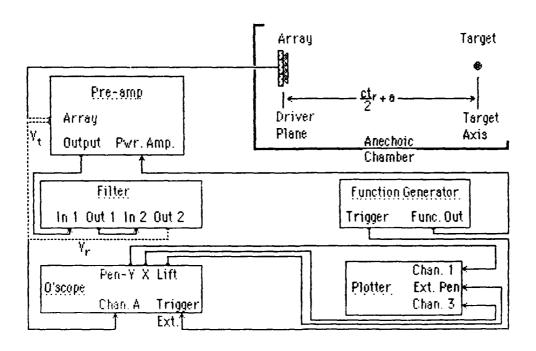


Figure 8: Data Acquisition Equipment Set-up

Four target spheres were available and measured for size. They included one hollow aluminum sphere (sphere number one) and three solid aluminum and brass spheres. Table (1) lists the data and the calculated values of  $A_{tqt} = \pi a^2$ , ka, and  $\sigma_b$ .  $\sigma_b$  was calculated from equation (86b).

Table 1: Target Sphere Dimensional Data and Normalized Backscattered Cross Section Calculational Results for Frequency of 5000 [Hz].

Target No.	Sphere Diameter [m]	Sphere Radius [m]	A <sub>tgt</sub> [m <sup>2</sup> ]	ka	<b>σ</b> <sub>b</sub>
1	.2546 ±.0006	.1273	.05091	11.62	.9142
2	.1013 ±.0001	.05065	.008060	4.619	1.000
3	.07634±.00003	.03817	.004577	3.479	1.051
4	.06240±.00008	.03120	.003058	2.843	.6836

Figure 9 shows the target spheres' theoretical normalized backscattered cross section distribution plotted on a graph generated by the backscattered cross section program of Appendix C.

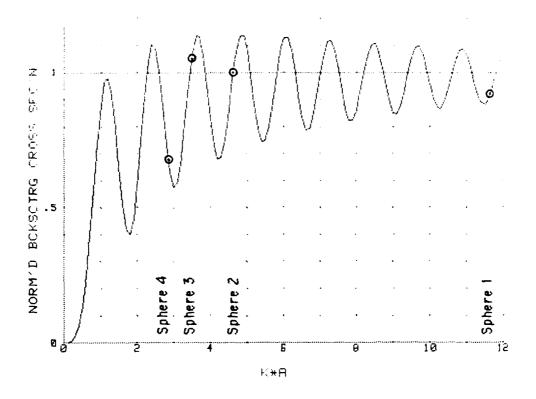


Figure 9: Target Spheres' Normalized Backscattered Cross Section Distribution as Calculated from Measured Diameters, Using an Assumption of Perfect Acoustic Hardness.

The target spheres were suspended in turn in the anechoic chamber. Placement was made using the criteria in Section D.1 of Chapter II. The range R from the array to a target was determined to be 5.30 [m].

Temperature  $T_{\rm c}$  and relative humidity Rh along the propagation path were determined with a Weathermeasure Temperature and Relative

Humidity Meter.  $T_c$  was usually about 21 [°c] with a relative humidity of approximately 50%. Atmospheric pressure P was read from a barogram supplied by the Naval Postgraduate School meteorological station. Specific  $T_c$ , Rh, and P data are furnished in Appendix E.

The unshrouded, close packed, hexagonal acoustic array of Moxcey [Ref. 8] was used to ensonify each target.

$$A = N_s \pi r_s^2 \tag{96}$$

determined the aperture area A of the array.  $N_s$  = 19 was the number of speakers in the array and  $r_s$  = .0381 [m] was the average speaker horn radius [Ref. 8].

Moxcey [Ref. 8] determined the half width of the array's main lobe to be 12°. The angular half widths of targets two, three and four were all less than 5% of the main lobe's half width. Target one's angular half width was 11.5% of the main lobe's half width.

A sinusoidal input from a Hewlett Packard 3314 A function generator drove the array via a pre-amplifier. Signal input from the HP3314A function generator to the pre-amplifier was set for 20 cycles at 5000 [Hz]. The voltage input to the pre-amplifier's power amplifier

terminal (see Figure 8) was 3.5 [V] for spheres two, three and four and 3.0 [V] for sphere one.

Twenty cycles is considerably less than the number of cycles expected in actual echosounder application. It was necessary to keep the pulse packet this small because the anechoic chamber's corner was only about 1.5 [m] from the target mounting location. Voltage input for sphere one was reduced to 3.0 [V] to avoid clipping of its greater return signal voltage.

Actual transmission voltage inputs  $V_t$  to the array from the pre-amplifier were measured with a Nicolet 3091 oscilloscope at the pre-amplifier's array terminal. The array was disconnected from the pre-amplifier for  $V_t$  measurements.  $V_t$  values are provided in Appendix E. They are also stored in the Trans30 and Trans35 subprograms of the acoustic array efficiency program in Appendix D.

The Nicolet oscilloscope was connected to a Hewlett Packard 7090 A measurement plotting system. This connection provided a hard copy plot of voltage vs. time traces.

 Filtering was done with a Wavetek Brickwall filter model 753 A to remove the low frequency normal modes of oscillation of the speaker drivers from the return signal. Filtering was accomplished at 0 [dB] gain and 5000 [Hz] for both low and high pass settings.

The processed return signal voltages  $V_r$  were displayed and measured on the Nicolet 3091 oscilloscope.  $V_r$  measurements were made at the Out 2 terminal of the Wavetek Brickwall filter (see Figure 8). They were taken at 10 [ $\mu$ S] intervals for two full cycles along the asymptotic region of the return signal trace.

The Nicolet 3091 oscilloscope was connected to the Hewlett

Packard 7090 A measurement plotting system. Figure 10 shows the

Hewlett Packard plotter's traces for target sphere two's return signal

voltages as printed and subsequently labeled. The figure is representative

of the oscilloscope traces for each target return. Data for all targets are

provided in Appendix E.

The time of return  $\mathbf{t_r}$  was determined as the time difference between the first positive maxima of the transmitted pulse packet and the first positive maxima of the target's return pulse packet.

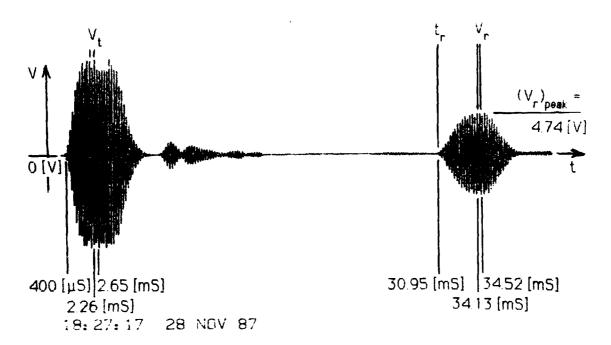
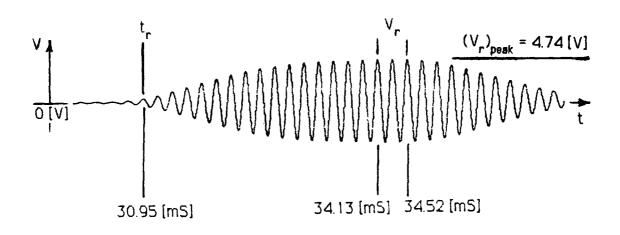


Figure 10, Sheet 1 of 2: Target Sphere Two's 5000 [Hz] Return Signal Trace



18:28:37 28 NOV 87

Figure 10, Sheet 2 of 2: Target Sphere Two's 5000 [Hz] Return Signal Trace, Scaled Down For Oscilloscope Measurement.

Noise voltage  $V_n$  measurements were made with the same measuring equipment setup used for  $V_r$  measurements. The target spheres were removed.  $V_n$  measurements were taken at a time of return  $t_r$  comparable to the time of measurement of  $V_r$ .

 $V_n$  measurements were actually made on occasions well separated chronologically from the  $V_r$  measurements. Therefore, they were proportionally corrected in line 2530 of the acoustic array efficiency program. The corrections account for the slight variations in input voltages and propagation path attenuations that occurred between the two measurement occasions.

Figure 11 shows a representative noise trace. Data from the noise measurements is presented in Appendix E and stored in the Noise30 and Noise35 subprograms of the acoustic array efficiency program in Appendix D.

Unless otherwise specified above the remaining data from the two way propagation path measurements are stored in the subprograms titled Sphere1, Sphere2, Sphere3 and Sphere4. All remaining data are also presented in Appendix E.

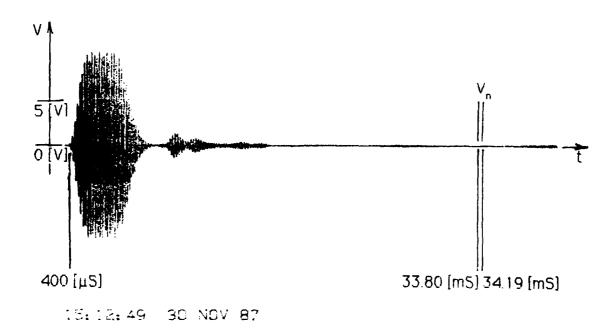


Figure 11: Representative 5000 [Hz] Noise Trace. 3.0 [V] Supplied by the HP3314A Function Generator.

# 2. One Way Propagation Path Measurements

One way propagation path data were collected 7 December. Measurements were used to calculate  $E_{\xi}G_{o}$  with equation (17).

Data from the one way propagation path measurements are provided in Appendix E. Data are also stored in the Gain30 and Gain35 subprograms of the acoustic array efficiency program of Appendix D.

For one way path measurements, i.e. for the determination of  $E_tG_0$ , an Ivie Electronics Inc. IE 30 A Audio Analyzer (Serial #805 A 426) was used to measure received target intensity I<sub>r</sub> in [dB]. Intensity measurements were referenced to  $10^{-12}$  [W/m²]. The acoustic signal supplied for this measurement was a continuous wave at 5000 [Hz]. Measurements were taken at the suspension site of the calibrating targets.

A Fluke 8060 A True RMS Multimeter was used to measure the root-mean-square voltage  $(V_{rms})_t$  supplied to the acoustic array at its input.  $(V_{rms})_t$  was squared to determine  $(V_{ms})_t$ .

The Weathermeasure temperature and relative humidity meter was unavailable for  $E_tG_0$  data collection. Hygrothermographs from the Naval Postgraduate School's meteorological station were used to estimate the anechoic chamber's relative humidity Rh and temperature  $T_c$  on 7 December. Estimates were based on comparisons to the hygrothermographs and measured Rh and  $T_c$  of 28 and 30 November.

### C. RESULTS

Data analysis was performed with the aid of the acoustic array efficiency program of Appendix D. Data used in calculations is furnished in Appendix E.

The product  $\sigma_b E_r E_t G_o$  was generated from two way popagation path data and application of equation (26). The average of calculated  $\sigma_b$ 's compared to the average of the experimental product  $\sigma_b E_r E_t G_o$  gives a normalizing product  $E_r E_t G_o$ . The experimental product  $\sigma_b E_r E_t G_o$  and the normalizing product  $E_r E_t G_o$  are used to calculate experimental  $\sigma_b$ 's. These results are supplied in Table 2.

Table 2: Target Sphere Experimental, Normalized Backscattered Cross Section Calculational Results.

Target No.	ka [m²]	α <sup>ρ</sup> Calc.q	<mark>տ</mark> եբել <sup>6</sup> օ	E <sub>r</sub> E <sub>t</sub> G <sub>o</sub>	Exp'l
1	11.62	.9142	55.90		.947
2	4.619	1.000	59.88		1.01
3	3.479	1.051	61.62		1.04
4	2.843	.6836	37.98		.644
Avg.		.912	53.85	59.0	

The experimental  $\sigma_b$  are plotted on a graph generated by the backscattered cross section program of Appendix C. This plot is Figure 12. It is similar to Figure 9 and gives  $\sigma_b$  as a function of ka. Note ka is labeled K\*A.

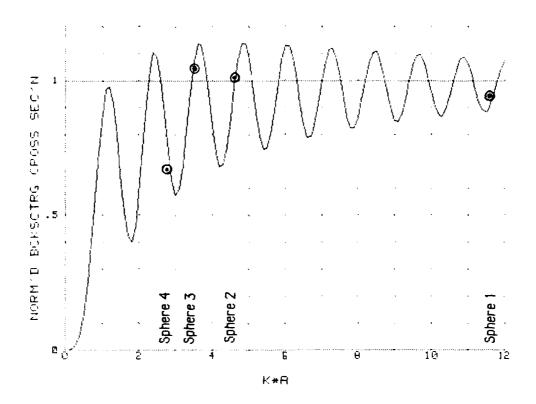


Figure 12: Target Spheres' Experimental, Normalized Backscattered Cross Section Distribution Compared to a Calculated Backscattered Cross Section Distribution Curve Generated by the Backscattered Cross Section Program of Appendix C.

As ka increases the experimental  $\sigma_b$  distribution rises from just below the calculated  $\sigma_h$  curve to just above it.

 $E_tG_0$  was determined using one way propagation path data and equation (17).  $E_tG_0$ , or  $E_t$ , was assumed to be the same for spheres two, three, and four because the supply voltage and target locations were the same. The angular half widths of target spheres two, three and four were between 4.6% and 2.8% of the echosounder's main lobe's half width.

Table 3: Acoustic Array Product  $E_tG_o$ Determined from One Way Propagation Path Measurements and Equation (17).

Target	HP3314A	$E_t^{}G_o^{}$
No.	Func'n Gen'r Supply Volt.[V]	
1	3.0	94.70
2	3.5	95.11
3	3.5	95.11
4	3.5	95.11

Calculated  $\sigma_b$ ,  $E_tG_o$  and two way propagation path data were used in equation (26) to determine  $E_r$ .  $\sigma_b$  values were calculated from the target

spheres' measured diameters, 2a.  $A_{tgt}$  was calculated as  $\pi a^2$ . Calculation results are furnished in Table 4.

Table 4:  $E_r$  and  $E_tE_r$ .  $E_r$  Determined From Two Way Propagation Path Data and Equation (26).  $E_tE_r$  Determined From Assumption of Efficiency Reciprocity.

Target No.	A <sub>tgt</sub> [m <sup>2</sup> ]	و <sup>4</sup> Cajc,q	$\sigma_b A_{tgt} [m^2]$	Er	E <sub>t</sub> E <sub>r</sub>
1	.05091	.9142	.04654	.6456	.4169
2	.008060	1.000	.008060	.6296	.3964
3	.004577	1.051	.004810	.6167	.3803
4	.003058	.6836	.002090	.5841	.3411

The  $E_r$ 's determined for Moxcey's unshrouded hexagonal array are in the range of .58 to .65. This is above the value range of 0.5  $\pm$  0.1 determined by Weingartner for his square acoustic array [Ref. 6].

Table 4 results were used to construct Figures 13 and 14. Figure 13 gives  $E_r$  as a function of the calculated, normalized backscattered cross section,  $\sigma_b$ . Figure 14 gives  $E_r$  as a function of the apparent target size,  $\sigma_b A_{tgt}$ .

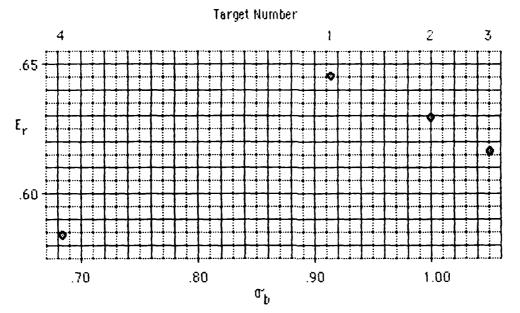


Figure 13: Acoustical to Electrical Power Conversion Efficiencies,  $E_{\bf r.}$  as a Function of the Normalized Backscattered Cross Sections,  $\sigma_b$ , Calculated from Measured Sphere Sizes.

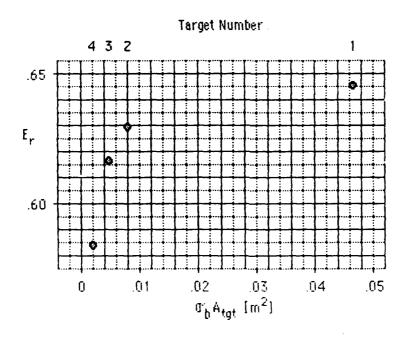


Figure 14: Acoustical to Electrical Power Conversion Efficiencies,  $E_r$  as a Function of the Apparent Target Sizes,  $\sigma_b A_{tgt}$ .

Recalling equation (20),

$$P_{tgt} = \left[\frac{P_{a}}{4\pi}\right] \left[e^{-\alpha R} G(\Omega)\right] \left[\frac{A_{tgt}}{R^2}\right], \qquad (20)$$

and equation (12),

$$P_{ra} = \frac{P_{tgt}}{4\pi R^2} \sigma_b e^{-\alpha R} A , \qquad (12)$$

serves as a reminder that the received acoustic power, P  $_{ra}$  , depends on the apparent target size,  $\sigma_b A_{tat}$ 

Figure 14's trend and the dependency of  $P_{ra}$  on  $\sigma_b A_{tgt}$  were used as suggestions for the construction of Figure 15. Figure 15 is a plot of  $E_r$ 

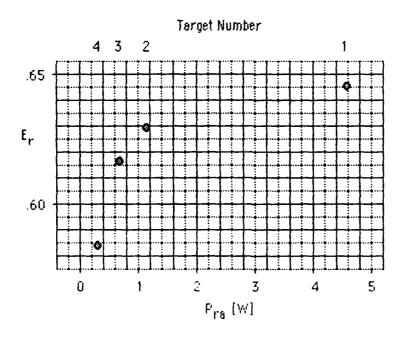


Figure 15: Acoustical to Electrical Power Conversion Efficiencies,  $E_{\rm r}$  as a Function of the Received Acoustic Powers,  $P_{\rm re}$ 

versus  $P_{ra}$   $P_{ra}$  was calculated using the definition (from equation (21))

$$P_{ra} = P_r / E_r \tag{97}$$

and the relationship (from equation (15))

$$P_r = (V_{ms})_r / Z$$
 (98)

Note Figure 15's trend is for  ${\rm E_r}$  to increase with an increase in  ${\rm P_{ra}}$  for the range of target sizes investigated.

#### IV. CONCLUSIONS AND RECOMMENDATIONS

#### A. CONCLUSIONS

The larger magnitudes of  $E_r$  determined for Moxcey's hexagonal array suggest it may be more efficient than Weingartner's square array. However, the drivers for each array are identical.

The 0.08 to 0.15 differences in efficiencies are more likely the result of the difference in calibration methods. Weingartner's measurements [Ref. 6] used steady state, continuous waves. The use of steady state, continuous waves allows the development of standing waves. This calibration investigation used pulsed waves. The use of pulsed waves did not provide sufficient time for standing waves to completely develop in the backscattered energy. [Ref. 26] This calibration method should provide more accurate efficiency values since it more nearly approximates the echosounder system's actual field operating configuration.

Additionally, steel braid and nylon lines were used to suspend the target spheres. These supports probably reflected enough acoustic energy to have a small influence on results. Other fixtures in the anechoic

chamber, such as support clamps and lights, may have also contributed small amounts of reflected energy to the returned signal.

Calibration of the computer controlled echosounder required determination of the product  $E_r E_t$ . The calibration process assumed  $E_r = E_t$ . The calibration results in Tables 3 and 4 for spheres two, three, and four indicate  $E_r$  varied with constant  $E_t G_0$ , or constant  $E_t$ . Such a variation negates the assumption of reciprocity for  $E_r$  and  $E_t$ .

Figure 15 suggests the acoustic array's efficiency  $E_r$  increased in response to an increase in  $P_{rs}$  and  $(V_{rms})_r$ .

The acoustic gain for spheres two, three and four was probably not constant, as assumed. However,  $G_0$  is the maximum for  $G(\Omega)$ , and  $E_r$  is inversely proportional to  $G_0$ , per equation (26). An averaging process accounting for the slight differences in gain encountered by spheres two, three and four would have increased the data points spread observed in Figures 14 and 15.

Additionally, the pre-amplifier's gain may not have been constant for the range of received voltage values encountered. This would have introduced either an apparent increase in  $E_{\rm r}$  or dampened an increase in  $E_{\rm r}$ 

with an increase in  $(V_{rms})_r$ . Such a gain variation could have effectively negated the assumption of efficiency reciprocity. However, this pre-amplifier is approximately 0.1% linear for less than 10 [V] peak [Ref. 26]. Received voltages in this investigation were all less than 10 [V].

### B. RECOMMENDATIONS

Because efficiency reciprocity is now in question it is necessary to determine  ${\sf G}_0$  by calculation or measurement. Such a determination would allow the desired calibration quantity  ${\sf E}_r{\sf E}_t$  to be written as

$$E_r E_t = E_r \frac{E_t G_0}{G_0} . (99)$$

One method of calculating  $G_0$  would involve integrating the acoustic array's calculated or measured intensity pattern (see Moxcey [Ref. 8]) for determination of an equivalent isotropic emitter. To determine  $G_0$  the centroid intensity of the array's emitted intensity pattern would be compared to that of the equivalent isotropic emitter. This would be repeated for each transmission power level of interest.

Calculation of  $G_0$  would allow a determination of  $E_t$  using  $E_tG_0$ . Such a determination of  $E_t$  would allow a reciprocity comparison to  $E_r$  and support a more accurate evaluation of  $E_rE_t$ .

The accuracy of  $E_r$  and  $E_tG_0$  could be further refined as follows.

Use a calibrating site that allows the use of a pulse train closer in length to actual field use.

Make target return and noise voltages and site intensity measurements as chronologically close as possible.

Measure the barometric pressure in the propagation path directly along with the relative humidity and temperature.

The received and transmitted electrical powers for  $\mathbf{E_r}$  and the transmitted electrical power for  $\mathbf{E_t}\mathbf{G_0}$  were all measured at the pre-amplifier end of the pre-amplifier/array transmission line. This included transmission line power losses due to impedance as efficiency decreases. The present calibration process includes the transmission line impedance as part of the array impedance.

The amount this impedance power loss actually affects  $\mathbf{E}_{\mathbf{r}}$  could be evaluated by determining the impedances seen in each direction at the

pre-amplifier's array terminal. This would be an opportune time to verify the validity of transmission line impedance reciprocity.

Use the Nicolet oscilloscope to record a triggered readout from a calibrated microphone suspended at the target site at  $45^{\circ}$  to the centroid's axis. This would allow  $E_tG_0$  to be determined for a pulse packet rather than for a continuous wave. Coupled with a proper calibrating site selection this technique should minimize interference problems such as anechoic chamber corner reflections.

A larger number of apparent target sizes spanning the desired range of apparent target sizes should be used to redetermine  $E_r$  for Moxcey's array. The  $E_r$  distribution should be examined for a smooth and asymptotic behavior. Comparative  $E_r$  values for Moxcey's array could be determined using the method outlined by Weingartner [Ref. 6].

Weingartner's square array's  $E_r$  could also be determined with this thesis' calibration process and compared to Weingartner's  $E_r$  value [Ref. 6].

The refined  $E_r$  curve in the now calibrated echosounder system should be used to develop an absolute  $C_T^2$  plot of an air mass. An established method such as a tower mounted vertical array of thermocouples would be

used simultaneously to generate a comparative absolute  $C_{\rm T}^{\,2}$  plot of the same air mass.

#### C. SUMMARY

Realizing the potential of the computer controlled echosounder for analyzing lower atmospheric turbulence required a calibration of the echosounder. This thesis described theory and software for performing calculations crucial to a backscattered cross section calibration of the echosounder. The backscattered cross section calibration was evaluated in this thesis.

 $E_r$ 's values indicate the evaluated calibration process requires some refinement. They also indicate the calibration method possesses sufficient merit to warrant further development and that the software performs as intended.

#### APPENDIX A

### ANECHOIC CHAMBER DESCRIPTION

The anechoic chamber is located in room 019 of building 232 at the Naval Postgraduate School. Acoustic research on sound sources, sound receivers and sound scatterers may be conducted in the chamber with a minimum of interference from wall reflection and external noise.

Wall reflection is minimized by 102 centimeter deep wedges made of P.F. 612 fiberglass. The wedges are attached to the walls, ceiling and floor of the chamber. Approximately 142 cubic meters of fiberglass is used to absorb 99% of incident sound energy for frequencies greater than 100 Hertz. External noise isolation is accomplished by separating the chamber's inner concrete block sides from the outer 12 inch concrete walls with a two inch thick lining of fiberglass and cork.

A usable region of approximately 8.2 meters by 4.3 meters by 3.4 meters is available. This region is floored by a grid of 225 wire cables, with each cable placed in a tension of 150 to 200 pounds per square inch. \*

<sup>\*</sup>This Appendix was compiled from the description posted by the entrance to the anechoic chamber instrumentation and control room.

# APPENDIX B

## DIFFERENTIAL SCATTERING CROSS SECTION PROGRAM

Examples of the output of this program are Figures 4 and 5, located in Chapter III.

10 20	! \$\$\$\$ !	\$	\$\$\$\$\$	555555555555555555555555555555555555555		
30 40	REM REM			RING OF PLANE ACOUSTIC WAVES		
50 60 70	REM REM REM	THIS PROGRAM CA DULATES AND CROSS SECTION AND/OR SCATTERIN		PHS THE DIFFERENTIAL SCATTERING DULUS FOR AN ACOUSTICALLY HARD		
80 90 100	REM REM REM	SPHERE OF DIAMETER D ENSONIFIE FREQUENCY F.	BY 6	AN INCIDENT PLANE WAVE OF		
130	REM !	+++++				
150 160 170	1++++	VARIABLE DECLARATIONS AND DEFI	1 T I O	NS ++++++++++++++++++++++++++++++++++++		
180 190	REAL	С	SPE	EED OF SOUND.		
200 210 220	REAL	•		EQUENCY OF THE EMITTER.		
230 240	REAL			DER OF THE SPHERE.		
250 260 270 280 290			AS A	A SUBROUTINE. IT IS USED A LOOP INDEX AND TO DESIGNATE E ARRAY ELEMENT FOR THE FUNCTION CORRESPONDING ORDER.		
300 310 320 330 340	REAL		DETE SCAT	(. SIGNIFICANT ORDER OF THE SUM TERMINING THE DIFFERENTIAL ATTERING CROSS SECTION (DSCS) D SCATTERING MODULUS (SM).		
350 360	REAL	К	THE	E WAVE NUMBER, 2*PI*FREQ/C.		
370 380 390 400	REAL		AND	A, THE ARGUMENT OF THE REGULAR D IRREGULAR SPHERICAL BESSEL NCTIONS AND THEIR DERIVATIVES.		

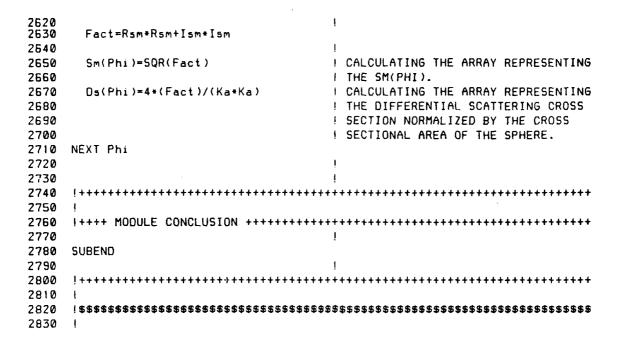
```
I ARRAY FOR THE REGULAR SPHERICAL
410
     REAL J(51)
420
                                    ! BESSEL FUNCTIONS (RSB) OF
430
                                    ! ARGUMENT KA AND ORDER Ø THROUGH
440
                                    ! LMAX.
450
                                    ! ARRAY FOR THE DERIVATIVES OF THE
460
     REAL Dj(51)
                                    ! RSB (THE DRSB).
470
480
490
     REAL Y(51)
                                    ! ARRAY FOR THE IRREGULAR SPHERICAL
500
                                    I BESSEL FUNCTIONS (ISB) OF
510
                                    ! ARGUMENT KA AND ORDER Ø THROUGH
520
530
                                    I ARRAY FOR THE DERIVATIVES OF THE
540
     REAL Dy(51)
                                    ! ISB (THE DISB).
550
560
570
     REAL P(51)
                                    I ARRAY FOR THE LEGENDRE
580
                                    1 POLYNOMIALS OF ARGUMENT COS(PHI).
590
                                    I THE ARRAY REPRESENTING THE
600
     REAL Sm(360)
                                    ! SCATTERING MODULUS (SM).
610
620
     REAL Ds(360)
                                    ! THE ARRAY REPRESENTING THE
630
540
                                    I DIFFERENTIAL SCATTERING CROSS
650
                                    ! SECTION NORMALIZED BY THE CROSS
                                    ! SECTIONAL AREA OF THE SPHERE
660
670
                                    I (DSCS).
680
                                    ! NOTE DS(PHI)=4*SM(PHI)^2/KA^2.
690
700
                                    ! ++++ NOTE ++++
710
                                    ! THE ARRAY DIMENSIONS (30+21) NEED
720
                                    I TO BE INCREASED TO ACCOMODATE
730
                                    ! KA>30.
740
750
     760
770
     780
790
800
     CALL Init(C,Freq,A,Lmax,Ka)
                                   ! INPUT AND CALCULATE REQUIRED
                                    ! PARAMETERS.
810
820
830
    CALL Difsctr(Lmax, ka, Uj(*), j(*), Dy(*), Y(*), P(*), Sm(*), Ds(*))
                                    ! CALCULATE THE SM(+) AND DS(+).
840
850
860
    CALL Outpt(C,Freq,A,Ka,Sm(*),Ds(*)) ! OUTPUT THE SM(*) AND/OR DS(*).
870
880
890
     900
```

```
920
930
 PRINTER IS 1
940
 PRINT
950
 PRINT "END"
               ! ADVISES THE USER OF THE MAIN
960
 PRINT
               ! PROGRAM'S CONCLUSION.
970
 END
980
990
 1000 !
1020 !
```

```
1030
    1050 SUB Init(C,Freq,A,Lmax,Ka)
        1060
        THIS MODULE REQUESTS THE APPROPRIATE INPUT VARIABLES TO DETERMINE
1070
    REM
1080
    REM
        C, FREQ, A, LMAX, AND KA.
       1090
    REM
1100
1110
    1120
1130
1140 REAL Temp
                              ! AMBIENT AIR TEMPERATURE IN DEG C.
1150
1160 REAL Circ
                              ! CIRCUMFERENCE OF THE TGT. SPHERE.
1170
1180 REAL Diam
                              ! DIAMETER OF THE TARGET SPHERE.
1190
1200
1220 1
1240
1250
                              ! INPUTS ARE REQUESTED ON THE CRT.
1260 PRINTER IS 1
1270
1280 Choice=0
                              ! *** DETERMINING C ***
1290 WHILE Choice <> 1 AND Choice <> 2
1300
     PRINT
     PRINT "DO YOU DESIRE TO INPUT"
1310
     PRINT " (1) THE SPEED OF ";
1320
     PRINT "SOUND,"
1330
     PRINT " OR (2) THE AMBIENT AIR ";
1340
1350
     PRINT "TEMPERATURE?"
1360
     PRINT
     INPUT Choice
1370
1380 END WHILE
1390
1400 IF Choice=1 THEN
                              I DIRECTLY ENTER C,
    PRINT "PLEASE ENTER C IN M/S."
1410
     INPUT C
1420
1430 END IF
1440
1450 IF Choice=2 THEN
                              ! OR CALCULATE C FROM TEMP.
1460
    PRINT "PLEASE ENTER TEMP IN ";
1470
     PRINT "DEGREES CELSIUS."
1480
     INPUT Temp
1490
     Temp=Temp+273.15
1500
     C=20.05 • SQR(Temp)
1510 END IF
                              ! *** DETERMINING FREQ ***
1520
1530 PRINT
                              ! DIRECTLY ENTER FREQ.
1540 PRINT "PLEASE ENTER THE ECHOSOUNDER";
1550 PRINT " FREQUENCY IN HZ."
1560 PRINT
1570 INPUT Freq
```

```
1580
1590 Choice=0
                               i *** DETERMINING KA ***
1600 WHILE Choice(>1 AND Choice(>2 AND Choice(>3
    PRINT
1610
     PRINT "DO YOU DESIRE TO INPUT "
1620
     PRINT " (1)THE SPHERICAL ";
1630
     PRINT "BESSEL ARGUMENT K*A."
1640
     PRINT " (2) THE SPHERE'S ";
1650
     PRINT "CIRCUMFERENCE,"
1660
     PRINT " OR (3) THE SPHERE'S ";
1670
     PRINT "DIAMETER?"
1680
     PRINT
1690
    INPUT Choice
1700
1710 END WHILE
1720
1730 IF Choice=1 THEN
                               ! DIRECTLY ENTER KA.
    PRINT "PLEASE ENTER K*A."
1740
     INPUT Ka
1750
1760 END IF
1770
                               1
                              I CALCULATE KA FROM THE
1780 IF Choice=2 THEN
1790 PRINT "PLEASE ENTER THE TARGET "; ! TARGET CIRCUMFERENCE,
    PRINT "CIRCUMFERENCE IN CM."
1800
1810 INPUT Circ
1820 Ka=Circ*(10^(-2))*Freq/C
1830 END IF
1840
1850 IF Choice=3 THEN
                               ! OR CALCULATE KA FROM THE
    PRINT "PLEASE ENTER THE TARGET "; ! TARGET DIAMETER.
1850
     PRINT "DIAMETER IN CM."
1870
     INPUT Diam
1880
1890
     Ka=Diam*(10^(-2))*PI*Freq/C
1900 END IF
1910
1920
                               ! *** DETERMINING REMAINING ***
1930
                               ! *** SYSTEM PARAMETERS ***
1940 K=2*PI*Freq/C
                               ! CALCULATE THE WAVE NUMBER.
                               I CALCULATE THE SPHERE RADIUS.
1950 A=Ka/K
1960 Lmax=INT(Ka+21)
                               ! MAX. SUMMATION ORDER CALCULATED.
1970
1980
2000 !
2020
                               1
2030 SUBEND
2040
2060 !
2080
```

```
2090
     2100
    SUB Difsctr(Lmax, Ka,Dj(*),J(*),Dy(*),Y(*),P(*),Sm(*),Ds(*))
2110
         2120
         THIS MODULE CALCULATES THE ARRAYS SM(*) AND DS(*) REPRESENTING THE
2130
    REM
2140 REM
         SCATTERING MODULUS AND THE DIFFERENTIAL SCATTERING CROSS SECTION
2150 REM
         NORMALIZED BY THE CROSS SECTIONAL AREA OF THE SPHERE, RESPECTIVELY.
2160 REM
         2170
2190
2200
2210 REAL Fac
                                 ! FACTOR COMMON TO RSM AND ISM.
2220
                                 ! THE REAL COMPONENT OF THE
2230 REAL Rsm
                                 ! SCATTERING MODULUS, SM.
2240
2250
2260 REAL Ism
                                 ! THE IMAGINARY COMPONENT OF THE
2270
                                 I SM.
2280
2290 REAL Fact
                                 ! FACTOR COMMON TO SM(PHI) & DS(PHI).
2300
                                 ! THE (POLAR) ANGLE OFF THE AXIS
2310 REAL Phi
                                 ! OF PROPAGATION OF THE INCIDENT
2320
                                -! PLANE WAVE, WITH ORIGIN AT THE
2330
                                 ! SPHERE CENTER. COS(PHI) IS THE
2340
                                 ! ARGUMENT OF THE LEGENDRE
2350
2360
                                 ! POLYNOMIALS P(+) OF ORDER 0
2370
                                 ! THROUGH LMAX.
2380
2390
2400
    2410
    2420
2430
2440
2450 CALL Drsb(Lmax, Ka, Dj(*), J(*))
                                 ! CALCULATE THE DERIVATIVES OF THE
                                 ! REGULAR SPHERICAL BESSEL FUNCTIONS
2460
2470
                                 ! OF ORDER Ø THROUGH LMAX.
2480 CALL Disb(Lmax, Ka, Dy(*), Y(*))
                                 ! CALCULATE THE DERIVATIVES OF THE
2490
                                 ! IRREGULAR SPHERICAL BESSEL FUNC'NS
2500
                                 ! OF ORDER Ø THROUGH LMAX.
2510 FOR Phi=0 TO 360
2520
2530
      CALL Leg(Lmax, Ka, Phi, P(*))
                                 : CALCULATE THE ARRAY P(*).
2540
2550
      Rsm=0
                                 ! CALCULATING THE REAL AND IMAGINARY
2560
      Ism=Ø
                                 ! COMPONENTS OF THE SM.
2570
      FOR L=0 TO Lmax
2580
       Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
2590
        Rsm=Rsm+(Fac+Dy(L))
2600
       Ism=Ism+(Fac+Dj(L))
2610
      NEXT L
```



```
2840 I
2860 SUB Drsb(Lmax,Ka,Dj(*),J(*))
2880 REM
     THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
2890 REM BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX.
2900 REM
     THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
2910 REM
     AND PREVIOUS ORDER.
2920 REM
     2930 !
2950
2960
                     1 CALCULATE THE RSB VALUES J(*).
2970 CALL Rsh(Lmax, Ka, J(*))
2980
                     ! ANGLE UNITS FOR DRSB CALC'NS.
2950 RAD
3000 Dj(0)=(-J(0)+COS(Ka))/Ka
                     ! CALCULATE THE INITIAL DRSB DJ(0).
3010 FOR L=1 TO Lmax
                     ! CALCULATE THE REMAINING DRSB.
3020
   Dj(L)=-((L+1)/Ka)*J(L)+J(L-1)
3030 NEXT L
3040
3050
                     !
3070 !
3090
3100 SUBEND
3110
3130 !
3150 !
```

```
3160
    -
3170
    3180 SUB Rsb(Lmax .Ka .J(*))
THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
3200 REM
        (RSB) OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX.
3210 REM
3220 REM
         CALCULATION USES THE CONTINUED FRACTION APPROACH.
3230 REM
         3240
3250
    3260
3270
3280 REAL R1
                                ! THE EVOLUING RATIO OF J(0)/J(L).
3290
                                ! THE EVOLVING NUMERATOR FACTOR USED
3300 REAL Numr
3310
                                I IN CALCULATION OF RL.
3320
3330 REAL Denr
                                ! THE EVOLVING DENOMINATOR FACTOR
3340
                                I USED IN CALCULATION OF RL.
3350
                                I EVOLVING SCALING EXPONENT FOR VERY
3360 REAL Scexp
                                I LARGE OR VERY SMALL VALUES OF RL.
3370
3380
3390 REAL Nflag
                                I 0 FLAG FOR NUMR.
3400
3410 REAL Dflag
                                I 0 FLAG FOR DENR.
3420
                                ! EVOLVING TERM USED IN THE
3430 REAL Bnm
3440
                                ! CALCULATION OF NUMR AND DENR.
3450
3460
    REAL Binc
                                ! INCREMENT USED IN THE EVOLUTION
3470
                                ! OF BNM.
3480
3490
3500
    3510
3520
    3530
3540
3550
    RAD
                                ! ANGLE UNITS FOR CALC'NS.
3560
3570
                                ! *** CALCULATE THE INITIAL ***
3580
    J(0)=(SIN(Ka))/Ka
                                ! *** RSB, J(0).
3590
3600
                                ! *** CALCULATING THE RATIO ***
3610 FOR L=1 TO Lmax
                                1 + + + J(0)/J(L).
3620
    RI=1.0
                                ! INITIALIZING RL.
3630
      Numr=0.
                                I INITIALIZING NUMR.
3640
      Denr=0.
                                ! INITIALIZING DENR.
3650
      Scexp=0.
                                I INITIALIZING SCEXP.
3660
      Nflag=0.
                                ! INITIALIZING NFLAG.
3670
      Dflag=0.
                                ! INITIALIZING DFLAG.
      Bnm=1.0/Ka
3680
                                ! INITIALIZING BNM.
3690
      Binc=2.0/Ka
                                ! CALCULATE BINC.
3700
```

```
I * CALCULATE THE L "NAKED"
3710
                                           ! * NUMR FACTORS OF J(0)/J(L). *
        FOR Ordent=1 TO L
3720
          Bnm=Bnm+Binc
                                           ! INCREMENT BNM.
3730
          IF Nflag=0 THEN
                                           ! CHECK NUMR=0 FLAG NOT SET:
3740
                                           ! UPDATE NUMR IF NUMR<>0.
            Numr=Bnm-Numr
3750
            IF Numr <> 0 THEN
3760
                                           ! UPDATE RL IF UPDATED NUMR<>0.
              R1=R1+Numr
3770
3780
              Numr=1.0/Numr
                                           ! PREPARE NUMR FOR NEXT EVOLUTION.
                                           ! CHECK SCALING OF UPDATED RL.
3790
3800
              IF ABS(R1)>10^250 THEN CALL Scale(R1.Scexp)
3810
            ELSE
              Nflag=1
3820
            END IF
3830
                                           ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
          ELSE
3840
                                           ! DEFERRED FOR ONE INCREMENT OF BNM.
3850
            Nflag=0
          END IF
3860
3870
        NEXT Ordent
3880
                                           I * CALCULATE THE REMAINING NUMR *
3890
                                           ! * AND DENR TERMS OF J(0)/J(L). *
3900
                                               NOTE WHEN NUMR=DENR THEN
3910
                                               RL=J(0)/J(L).
        WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
3920
          Bnm≃Bnm+Binc
                                           I INCREMENT BNM.
3930
3940
          IF Nflag=0 THEN
                                           ! CHECK NUMR=0 FLAG NOT SET;
3950
3960
                                           ! UPDATE NUMR IF NUMR<>0.
            Numr=Bnm-Numr
            IF Numr<>0 THEN
3970
              R1=R1 *Numr
                                           I UPDATE RL IF UPDATED NUMR<>0.
3980
                                           I PREPARE NUMR FOR NEXT EVOLUTION.
              Numr=1.0/Numr
3990
                                           ! CHECK SCALING OF UPDATED RL.
4000
4010
              IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
4020
            ELSE
4030
              Nflag=1
4040
            END IF
          ELSE
                                           ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
4050
                                           ! DEFERRED FOR ONE INCREMENT OF BNM.
4060
            Nflag=0
          END IF
4070
4080
4090
          IF Dflag=0 THEN
                                           ! CHECK DENR-0 FLAG NOT SET;
4100
            Denr=Bnm-Denr
                                           ! UPDATE DENR IF DENR<>0.
            IF Denr<>0 THEN
4110
4120
              R1=R1/Denr
                                           ! UPDATE RL IF UPDATED DENR<>0.
4130
              Denr=1.0/Denr
                                           ! PREPARE DENR FOR NEXT EVOLUTION.
4140
                                           I CHECK SCALING OF UPDATED RL.
4150
              IF ABS(R1)>10^2S0 THEN CALL Scale(R1,Scexp)
4160
            ELSE
4170
              Dflag=1
4180
            END IF
4190
          ELSE
                                           I IF DFLAG<>0 THE PREVIOUS STEPS ARE
4200
            Oflag=0
                                           ! DEFERRED FOR ONE INCREMENT OF BNM.
          END IF
4210
4220
        END WHILE
4230
4240
```

```
4250
4260 	 J(L)=J(0)/(R1*10^{(Scexp)})
             ! *** J(L) DETERMINED. ***
4270
4280 NEXT L
4290
4300
4310
 4320
 4330
4340
4350 SUBEND
4360
4380 !
4400 !
```

```
4410 !
4430 SUB Scale(R1,Scexp)
THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
4450 REM
   ONGOING CALCULATION OF RL.
4460 REM
4470 REM
    4480 |
4500
4510
4520 IF R1>10^250 THEN
  R1=R1*10^(-250)
4530
4540
   Scexp=Scexp+250
4550 END IF
4560 IF R1<10^(-250) THEN
4570
  R1=R1 * 10^250
  Scexp=Scexp-250
4580
4590 END IF
4600
4610
4630 !
4650
               ١
4660 SUBEND
4670
4690
4710 1
```

```
4720 !
4740 SUB Disb(Lmax, Ka, Dy(*), Y(*))
     4750 REM
     THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
4760 REM
4770 REM
     BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX. THE
4780 REM
     CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
4790 REM
     PREVIOUS ORDER.
4800 REM
     4810 !
4830
4840
                     I CALCULATE THE ISB VALUES.
4850 CALL Isb(Lmax, Ka, Y(*))
4860
4870 RAD
                     ! ANGLE UNITS FOR CALC'NS.
4880 Dy(0)=(-Y(0)+SIN(Ka))/Ka
                     ! THE INITIAL ISB DERIVATIVE.
4890 FOR L=1 TO Lmax
                     I THE REMAINING ISB DERIVATIVES.
   Dy(L)=-((L+1)/Ka)*Y(L)+Y(L-1)
4900
4910 NEXT L
4920
4930
4950 !
4970
                     1
4980 SUBEND
4990
                     - )
5010
5020
  5030 1
```

```
5040 !
5060 SUB Isb(Lmax,Ka,Y(*))
5070 REM
     THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
5080 REM
     OF ARGUMENT KA AND ORDER L THROUGH LMAX. THE CALCULATION IS
5090
  REM.
5100
  REM
     ACCOMPLISHED BY FORWARD RECURSION.
5110
  REM
     5120
5140
5150
5160 RAD
                   ! ANGLE UNITS FOR CALC'NS.
5170 Y(0)=~CGS(Ka)/Ka
                   ! INITIAL ISB'S CALCULATED.
5180 Y(1)=(Y(0)-SIN(Ka))/Ka
5190 FOR L=2 TO Lmax
                   ! REMAINING ISB'S CALC'D.
5200
   Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
5210 NEXT L
5220
5230
5250
  5260
5270
5280 SUBEND
5290
                  · 1
5310 1
5330 !
```

```
5340 !
5360 SUB Leg(Lmax, Ka, Phi, P(*))
THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT
5380 REM
5390 REM
    COS(PHI) AND ORDERS @ THROUGH LMAX. CALCULATION IS ACCOMPLISHED
5400 REM
    BY FORWARD RECURSION.
5410 REM
     5420
5440
5450
5460 DEG
                   ! ANGLE UNITS FOR CALC'NS.
5470 X=COS(Phi)
                   ! ARGUMENT OF THE LEG. POL'S.
5480
5490 P(0)=1
                   I INITIAL LEG. POL'S. CALC'D.
5500 P(1)=X
5510 FOR L=2 TO Lmax
                   ! REMAINING LEG. POL'S. CALC'D.
5520
  P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
5530 NEXT L
5540
S550
                   1
5570 !
5590
5600 SUBEND
5610
5620
  5630
5640
  5650 !
```

```
5680 SUB Outpt(C,Freq,A,Ka,Sm(*),Os(*))
5700 REM
        THIS MODULE CONTROLS THE OUTPUT OF THE CALCULATED DSCS AND SM IN
5710 REM GRAPHICAL PRESENTATIONS.
5720
    5730
5740
    5750
5760 REAL Desire
                                ! INTERACTIVE LOOP CONTROL.
5770
5780 REAL Rpt
                                ! POLARPLT SCALING CONTROL; A
5790
                                ! REPETITION INDEX.
5800
5820 !
5850 Desire=0
5860 WHILE Desire(>1 AND Desire(>2 AND Desire(>3
5870
      PRINT "DO YOU DESIRE"
      PRINT "
              (1) A PLOT OF THE DSCS,"
5880
      PRINT "
              (2) A PLOT OF THE SM."
5890
      PRINT " OR (3) BOTH?"
5900
     INPUT Desire
5910
5920 END WHILE
5930 PRINTER IS 701
5940
5950 WHILE Desire<4
5960
    IF Desire=1 OR Desire=3 THEN
5970
       FOR Rpt=1 TO 2
5980
        CALL Polarpit(Ds(*),Rpt) ! PLOTTING THE DSCS.
5990
         PRINT
6000
       NEXT Rpt
    END IF
6010
6020
     IF Desire=2 THEN
6030
      CALL Recplt(Sm(*))
                               ! PLOTTING THE SM.
6040
6050
       PRINT
      END IF
6060
6070
     PRINT " ","C =";C;"M/S."
PRINT " ","F =";Free:"47 "
6080
                               ! ECHO SIGNIFICANT CALCULATED VALUES
     PRINT " ","F =";Freq;"HZ."
PRINT " ","A =";A;"M."
PRINT " ","K*A =";Ka;"."
PRINT " ","NORMALIZED ";
6090
                               ! AND INPUT PARAMETERS.
6100
6110
6120
6130
      PRINT "BCKSCTR = ":Ds(180)
6140
      PRINT CHR$(12)
                                ! PRINTER FORM FEED.
6150
6160
    IF Desire=1 OR Desire=2 THEN Desire=4
6170
    IF Desire=3 THEN Desire=2
6180 END WHILE
6190
```

6200	l l
6210	
6220	· I
6230	!++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++
6240	+
6250	SUBEND
6260	1
6270	<u> </u>
6280	· ·
6290	! \$
6300	1

```
6310 !
6320
   6330 SUB Polarplt(Ds(*),Rpt)
       6340 REM
       THIS MODULE MAPS THE CALCULATED, NORMALIZED DSCS ARRAY DS(*) ON
6350 REM
       LINEAR POLAR PLOTS. THE FIRST PLOT INCLUDES THE ENTIRE DSCS.
6360 REM
       WHILE THE SECOND PLOT SHOWS THE SCATTERING DETAIL AROUND THE
6370 REM
6380 REM
       NORMALIZED RANGE. PLOT SELECTION IS GOVERNED BY A REPETITION
6390 REM
       INDEX, Rpt, AND IS CONTROLLED IN THE CALLING MODULE (OUTPT).
6400 REM
       6410 !
6430
6440
6450 REAL Xgumax
                            ! THE MAXIMUM ABSCISSA VALUE IN
6460
                            ! GRAPHICS DISPLAY UNITS.
6470
                            ! THE MAXIMUM ORDINATE VALUE IN
6480 REAL Ygumax
6490
                            ! GRAPHICS DISPLAY UNITS.
6500
6510 REAL Gumax
                            ! THE MINIMUM OF XGUMAX AND YGUMAX;
6520
                            ! USED TO DETERMINE PLOTTING AREA.
6530
6540 REAL Dtmx
                            ! THE MAX. VALUE OF THE ARRAY DS(*).
6550
6560 REAL Endpt
                            ! THE MAX. RADIAL SCALE VALUE IN
6570
                            ! USER DEFINED UNITS.
6580
6590 REAL Ring
                            ! GRID RING SPACING IN USER UNITS.
6600
6610 REAL Tick
                           ! RADIAL AXIS TICK MARK SPACINGS IN
6620
                            ! USER DEFINED UNITS.
6630
6640
6660
6680
6690 GINIT
6700 PLOTTER IS CRT, "INTERNAL"
6710 GRAPHICS ON
6720 GCLEAR
6730
```

```
6750
    6750
6770
6780
6790 Xgumax=100+MAX(1,RATIO)
6800 Ygumax=100+MAX(1,1/RATIO)
6810 Gumax=MIN(Xgumax, Ygumax)
6820
6830 CSIZE 3
                                       ! ** LABELLING MAJOR RADIALS **
6840 LINE TYPE 1
6850 MOVE Gumax, Gumax/2
6860 LORG 8
6870 LABEL "0"
6880 MOVE Gumax/2, Gumax
6890 LORG 6
6900 LABEL "PI/2"
6910 MOVE 0,6umax/2
6920 LORG 2
6930 LABEL "PI"
6940 MOVE Gumax/2,0
6950 LORG 4
6960 LABEL "3*PI/2"
                                       ! ** DEFINES THE TOTAL PLOTTING **
6970
6980 VIEWPORT 5,Gumax-5,5,Gumax-5
                                       ! ** AREA IN GRAPH UNITS.
6990
7000
                                       I ** SCALING THE GRAPH TO **
7010
                                       ! ** USER UNITS.
7020 Dtmax=0
                                       I DETERMINING MAX. RADIUS VALUE
7030 IF Rpt=1 THEN
                                       I REQ'D FOR THE PLOT.
7040
     FOR I=0 TO 360
7050
         IF Dtmx(Ds(I) THEN Dtmx=Ds(I)
7060
       NEXT I
7070 END IF
7080 Endpt=INT(Dtmx+1)
7090 IF Endpt<2 THEN Endpt=2
7100 SHOW -Endpt Endpt -Endpt Endpt
                                       ! ISOTROPICALLY SCALES THE GRAPH.
7110
7120 LINE TYPE 4
                                       ! ** MAJOR RADIALS CONSTRUCTION **
7130 Ring=INT(Endpt/5+1)
7140 Tick=.2*Ring
7150 AXES Tick, Tick, 0,0,5,5,2
7160
7170 LINE TYPE 3
                                       ! ** RANGE RING CONSTRUCTION **
7180 DEG
7190 LORG 7
7200 FOR R=1 TO Endpt/Ring
7210
       LINE TYPE 1
       MOVE 0 ,R+Ring
7220
7230
       LABEL R*Ring
                                      ! LABELLING THE RANGE.
7240
       LINE TYPE 3
7250
       FOR Angle=0 TO 360
                                      ! DRAWING THE RING.
7260
         PLOT R*Ring*COS(Angle),R*Ring*SIN(Angle)
7270
       NEXT Angle
7280
       PENUP
7290 NEXT R
```

```
7300
7310 FOR Angle=30 TO 150 STEP 30
                    | ** 30 DEGREE RADIALS **
                    ! ** CONSTRUCTION.
7320
   IF Angle<>90 THEN
    PLOT Endpt*COS(Angle), Endpt*SIN(Angle), -2
7330
    DRAW Endpt * COS(180+Angle), Endpt * SIN(180+Angle)
7340
7350
   END IF
7350 NEXT Angle
7370 PENUP
7380
                    ļ
7390
7400
  7410
  7420
7430
7440 LINE TYPE 1
7450 DEG
7460 FOR I=0 TO 360
                   I PLOTTING RECTANGULAR COORDINATES
7470
  PLOT Ds(I)*COS(I),Ds(I)*SIN(I) ! IN POLAR FORMAT.
7480 NEXT I
7490
7510 !
7530
                    ł
7540 DUMP GRAPHICS #701
7550
7560
  7570
  ļ
7580
  7590
7600 GRAPHICS OFF
7610 SUBEND
7620
                    1
7640 1
7660 !
```

```
7670
   7680
   SUB Recplt(Sm(*))
7690
       <del></del>
7700
   REM
       THIS MODULE MAPS THE CALCULATED SCATTERING MODULUS ARRAY SM(+) ON
7710
   REM
7720 REM
       A SEMI-LOG PLOT.
7730 REM
       7740
   7750
7760
7770
7780 REAL Xgumax
                          ! THE MAXIMUM ABSCISSA VALUE IN
7790
                          I GRAPHICS DISPLAY UNITS.
7800
7810 REAL Ygumax
                          ! THE MAXIMUM ORDINATE VALUE IN
                          ! GRAPHICS DISPLAY UNITS.
7820
7830
7840 REAL Dtmx
                          ! THE MAX. VALUE OF THE ARRAY SM(+).
7850
                          ! THE MIN. VALUE OF THE ARRAY SM(+).
7860 REAL Dtmn
7870
7880 REAL Mxy
                          ! THE MAX. ORDINATE SCALE VALUE IN
7890
                          ! USER DEFINED UNITS.
7900
7910 REAL Mny
                          ! THE MIN. ORDINATE SCALE VALUE IN
7920
                          ! USER DEFINED UNITS.
7930
7940
7950
   7960
7970
   7980
7990 GINIT
8000 PLOTTER IS CRT, "INTERNAL"
8010 GRAPHICS ON
8020 GCLEAR
8030
8050 !
8070
8080
8090 Xgumax=100*MAX(1,RATIO)
8100 Ygumax=100+MAX(1,1/RATIO)
8110
8120 CSIZE 4
8130
8140 LORG 4
                          ! ** LABELLING THE ABSCISSA AXIS **
8150 MOVE Xgumax/2,0
8160 LABEL "PHI (DEGREES)"
```

```
8170
8180 DEG
                                        ! ** LABELLING THE ORDINATE AXIS **
8190 LDIR 90
8200 LORG 6
8210 MOVE 0, Ygumax/2
8220 LABEL "SCTRG MODULUS"
8230 LDIR 0
                                       ! ** GRAPH CONSTRUCTION **
8240
8250 VIEWPORT 15, Xgumax-5, 10, Ygumax-5
                                        I DEFINES THE TOTAL PLOTTING AREA
8260
                                        ! IN GRAPH UNITS.
8270
8280 Dtmx=0
                                        ! ** ORDINATE USER UNITS **
8290 Dtmn=Sm(180)
                                        ! ** DETERMINED. **
8300 FOR I=0 TO 180
8310 IF Dtmx(Sm(I) THEN Dtmx=Sm(I)
     IF Dtmn>Sm(I) THEN Dtmn=Sm(I)
8320
8330 NEXT I
8340 Mxy=INT(LGT(Dtmx))+1
8350 Mny=INT(LGT(Dtmn))
8360
8370 WINDOW 0,180,Mny,Mxy
                                       ! ANISOTROPICALLY SCALES THE GRAPH
8380
                                       ! TO USER UNITS.
                                       ! PLOTS THE AXES.
8390 AXES 30,1,0,Mny,1,1,1.5
8400
8410 LINE TYPE 3
                                      I * PLOTS THE HORIZONTAL
8420 FOR I=Mny TO Mxy
     FOR J=1 TO 9
                                        ! * LOGARITHMIC GRID LINES I.A.W. *
8430
     MOVE Ø,I+LGT(J)
                                       ! * THE VERTICAL SCALE.
8440
       PLOT 180,I+LGT(J),-1
8450
       PENUP
8460
8470
     NEXT J
8480 NEXT I
8490
                                      ! * PLOTS THE VERTICAL GRID LINES *
8500 FOR I=60 TO 180 STEP 60
8510 MOVE I Mny
                                       1 * I.A.W. THE HORIZONTAL SCALE. *
       PLOT I, Mxy, -1
8520
8530
      PENUP
8540 NEXT I
                                        ļ
8550
8560 CLIP OFF
8570 LINE TYPE 1
8580
                                        I ** LABELLING THE ABSCISSA SCALE **
8590 LORG 6
8600 FOR I=0 TO 180 STEP 30
8610
     MOUE I,Mny
8620
      LABEL I
8630 NEXT I
8640
                                        ! ** LABELLING THE ORDINATE SCALE **
8650 LORG 8
8660 FOR I=Mny TO Mxy
8670 MOVE 0,1
8680 LABEL 10^I
                                        ! POWER OF 10 LABELS.
8690
      MOVE 0.1+LGT(5)
8700 LABEL (10^(I+1))/2
                                        ! HALF THE NEXT POWER OF 10 LABELS.
8710 NEXT I
```

8720		
8730	CLIP ON	
8740		
8750		
8760	1++++++++++++++++++++++++++++++++++++++	+++++++++++++++++++++++++++++++++++++++
8770	!	
8780	!++++ PLOTTING THE CALCULATED SCATTER	RING MODULUS +++++++++++++++++++++++++++++++++++
8790		
8800	LINE TYPE 1	THIS PLOT IS FOR THE PLANE WAVE
8810		INCIDENT FROM THE RIGHT (0 DEG'S).
8820	PLOT I LGT(Sm(I+180))	USE SM(I) IN PLACE OF SM(I+180)
8830	NEXT I	FOR A PLANE WAVE INCIDENT FROM THE
B840	•	LEFT (180 DEG'S).
8850	!	
B860	<u> </u>	+++++++++++++++++++++++++++++++++++++++
8870	1	
8880	!++++ PRINTING THE PLOT ++++++++++	+++++++++++++++++++++++++++++++++++++
8890	1	
8900	DUMP GRAPHICS #701	
8910		
8920	!++++++++++++++++++++++++++++++++++++++	+++++++++++++++++++++++++++++++++++++++
8930		
8940	1++++ MODULE CONCLUSION +++++++++	+++++++++++++++++++++++++++++++++++++++
9950		
8960	GRAPHICS OFF	
8970	SUBEND	
8980		
8990	! + + + + + + + + + + + + + + + + + + +	+++++++++++++++++++++++++++++++++++++++
9000	1	
9010	\$	;\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$ <b>\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$\$</b>

# APPENDIX C

## BACKSCATTERED CROSS SECTION PROGRAM

Examples of the output of this program are Figures 6 and 7, located in Chapter III.

1 <b>0</b> 20	18888	56\$	\$
30 40	REM REM		SCATTERING OF PLANE ACOUSTIC WAVES
50 60 70	REM REM	CROSS SECTION FOR ACOUSTICALLY	RAPHS THE NORMALIZED, BACKSCATTERED HARD SPHERES OF A SELECTED RANGE
80 90 1 <b>0</b> 0	REM REM REM	OF DIAMETERS D ENSONIFIED BY AN	N INCIDENT PLANE WAVE.
110 120 130	REM REM !	***************************************	***************************************
140 150 160	l++++	MAIN PROGRAM VARIABLE DECLARATI	ION AND DEFINITION ++++++++++++++++++++++++++++++++++++
170 180 190 200	REAL I		THE ARRAY REPRESENTING THE NORMALIZED, BACKSCATTERED CROSS SECTION (NBCS).
210 220	REAL I	<b>(</b>	THE WAVE NUMBER, 2*PI*FREQ/C.
230 240	REAL A	<b>A</b> !	RADIUS OF THE SPHERE.
250 260 270 280	REAL I		K.A., THE ARGUMENT OF THE REGULAR AND IRREGULAR SPHERICAL BESSEL FUNCTIONS AND THEIR DERIVATIVES.
290 300	REAL I	Kamın !	INTEGER OF THE MIN. VALUE OF KA.
310 320	REAL I	Kamax	INTEGER OF THE MAX. VALUE OF KA.
330 340 350	REAL !	5tp !	KA INCREMENT.
360 370	!++++ !	*********	· ++++++++++++++++++++++++++++++++++++

```
380
  390
  CALL Boksctr(Nbs(*), Kamin, Kamax, Stp) | CALCULATE THE NBS(*).
400
410
  CALL Recplt(Nbs(*), Kamin, Kamax, Stp) ! OUTPUT THE NBS(*).
420
430
  440
450
460
  470
480
  PRINTER IS 701
490
  PRINT CHR$(12)
500
510
520
530
540
  550
```

560	į	1				
570	! \$\$\$	!\$				
580	SUB	SUB Bcksctr(Nbs(*),Kamin,Kamax,Stp)				
590	REM	REM ++++++++++++++++++++++++++++++++++++				
600	REM	The state of the s				
610	REM		RMALIZED BY THE CROSS SECTIONAL AREA			
620	REM					
630	REM	+++++++++++++++++++++++++++++++++++++++	+++++++++++++++++++++++++++++++++++++			
640	1	. MODULE HARTAN, E. DER. AGATTANA A.				
650	!+++	+ MODULE VARIABLE DECLARATIONS A	ND DEFINITIONS ++++++++++++++++++++++++++++++++++++			
660 670			!			
680	REAL		! CORDER OF THE SUMMERS ON THE			
690	KERL		ORDER OF THE FUNCTION CALCULATED			
700			I IN A SUBROUTINE. IT IS USED			
710			AS A LOOP INDEX AND TO DESIGNATE			
720		•	THE ARRAY ELEMENT FOR THE FUNCTION			
730			OF CORRESPONDING ORDER.			
740	REAL		MAX. SIGNIFICANT ORDER OF THE SUM			
750		<del>-</del>	! DETERMINING THE DIFFERENTIAL			
760			SCATTERING CROSS SECTION (DSCS)			
770			! AND SCATTERING MODULUS (SM).			
780		· ·	(			
790	REAL		ARRAY FOR THE REGULAR SPHERICAL			
800			BESSEL FUNCTIONS (RSB) OF			
810			! ARGUMENT KA AND ORDER Ø THROUGH			
820			! LMAX.			
830						
840	REAL	D <sub>J</sub> (51)	ARRAY FOR THE DERIVATIVES OF THE			
850			RSB (THE DRSB).			
860		•	!			
870	REAL	Y(51)	ARRAY FOR THE IRREGULAR SPHERICAL			
880		!	BESSEL FUNCTIONS (ISB) OF			
890		!	! ARGUMENT KA AND ORDER Ø THROUGH			
900			LMAX.			
910		_ !	· ·			
920	REAL		ARRAY FOR THE DERIVATIVES OF THE			
930		!	ISB (THE DISB).			
940	054	1				
950	REAL		ARRAY FOR THE LEGENDRE			
960		!	POLYNOMIALS OF ARGUMENT -1.			
970	DC 41					
980	REAL		THE REAL COMPONENT OF THE			
990		<u>!</u>	SCATTERING MODULUS.			
1000	KEAL	! !	THE IMACINARY COMPONENT OF THE			
1020	KENL	•	THE IMAGINARY COMPONENT OF THE			
1030		!	SCATTERING MODULUS.			
1040	REAL	Fac	EACTOD COMMON TO FORM AND TON			
1050	NENE	, 40	FACTOR COMMON TO REM AND ISM.			
1060			++++ NOTE ++++			
1070			THE ARRAY DIMENSIONS (30+21) NEED			
1080			TO BE INCREASED TO ACCOMODATE			
1090			KAMAX>30.			
		·				

```
1100
1110
1120
1130
1140
    1150
1160
1170 PRINTER IS 1
    INPUT Kamin,Kamax
                                 ! DETERMINE DESIRED RANGE OF KA
1180
1190 Kamin=INT(Kamin)
                                 ! VALUES.
1200 Kamax=INT(Kamax)
1210 Stp=.1
1220
1230 FOR Ka=Kamin+Stp TO Kamax STEP Stp
1240
     Lmax=INT(Ka+21)
1250
      CALL Drsb(Lmax,Ka,Dj(*),J(*))
                                 ! CALCULATE THE DRSB OF ARGUMENT KA
1260
                                 ! AND ORDERS @ THROUGH LMAX.
1270
                                ! CALCULATE THE DISB OF ARGUMENT KA
      CALL Disb(Lmax,Ka,Dy(*),Y(*))
1280
                                 ! AND ORDERS Ø THROUGH LMAX.
1290
      CALL Leg180(Lmax,Ka,P(*))
                                ! CALCULATE P(*) OF ARGUMENT -1 AND
1300
                                 ! ORDERS Ø THROUGH LMAX.
1310
      R5m=0
                                 ! CALCULATING THE REAL AND IMAGINARY
1320
      Ism=0
                                ! COMPONENTS OF THE SCATTERING
1330
      FOR L=0 TO Lmax
                                 I MODULUS.
1340
       Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
1350
       Rsm=Rsm+(Fac+Dy(L))
1360
       Ism=Ism+(Fac+Dj(L))
1370
      NEXT L
1380
                                 ! CALCULATING THE ARRAY REPRESENTING
1390
                                 ! THE NBCS.
      Nbs((Ka~Kamin)/Stp)=4*(Rsm*Rsm+Ism*Ism)/(Ka*Ka)
1400
1410
1420 NEXT Ka
1430
1440
1450
    1460
1470
    1480
1490
    SUBEND
1500
1510
1520
1530
    1540
```

```
1550
1570 SUB Drsb(Lmax,Ka,Dj(*),J(*))
1590 REM
     THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
1600 REM
     BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX
     THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
1610 REM
     AND PREVIOUS ORDER.
1620 REM
     1630 REM
1640 |
1660
1670
1680 CALL Rsb(Lmax, Ka, J(*))
                     ! CALCULATE THE RSB VALUES J(+).
1690
1700 RAD
                     ! ANGLE UNITS FOR DRSB CALC'NS.
1710 D_{j}(0)=(-J(0)+COS(Ka))/Ka
                     ! CALCULATE THE INITIAL DRSB DJ(0).
1720 FOR L=1 TO Lmax
                     ! CALCULATE THE REMAINING DRSB.
   Dj(L)=-((L+1)/Ka)*J(L)+J(L-1)
1730
1740 NEXT L
1750
1760
                     1
1770
   1780
  ļ
1800
1810 SUBEND
1820
1840 !
1860 !
```

```
1870 1
1890 SUB Rsb(Lmax, Ka, J(*))
        1900 REM
        THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
1910 REM
        (RSB) OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX.
1920 REM
        CALCULATION USES THE CONTINUED FRACTION APPROACH.
1930 REM
        1940 REM
1950 !
1970
1980
1990 REAL R1
                               I THE EVOLVING RATIO OF J(0)/J(L).
2000
2010 REAL Numr
                               ! THE EVOLVING NUMERATOR FACTOR USED
2020
                               ! IN CALCULATION OF RL.
2030
2040 REAL Denr
                               ! THE EVOLVING DENOMINATOR FACTOR
                               I USED IN CALCULATION OF RL.
2050
2060
                               ! EVOLVING SCALING EXPONENT FOR VERY
2070 REAL Scexp
                               I LARGE OR VERY SMALL VALUES OF RL.
2080
2090
                               ! 0 FLAG FOR NUMR.
2100 REAL Nflag
2110
2120 REAL Dflag
                               ! Ø FLAG FOR DENR.
2130
2140 REAL Bom
                               ! EVOLVING TERM USED IN THE
2150
                               ! CALCULATION OF NUMR AND DENR.
2160
                               ! INCREMENT USED IN THE EVOLUTION
2170 REAL Binc
                               I OF BNM.
2180
2190
2200
2220
2230
    2240
2250
2260 RAD
                               ! ANGLE UNITS FOR CALC'NS.
2270
2280
                               ! *** CALCULATE THE INITIAL ***
2290 J(0)=(SIN(Ka))/Ka
                               ! *** RSB, J(0).
2300
                               I *** CALCULATING THE RATIO ***
2310
2320 FOR L=1 TO Lmax
                               ! *** J(0)/J(L).
2330
    R1=1.0
                               ! INITIALIZING RL.
2340
     Numr=0.
                               ! INITIALIZING NUMR.
2350
     Denr=0.
                               ! INITIALIZING DENR.
2360
      Scexp≈0.
                               ! INITIALIZING SCEXP.
2370
      Nflag=0.
                               ! INITIALIZING NFLAG.
2380
     Dflag=0.
                               I INITIALIZING DFLAG.
2390
      Bnm=1.0/Ka
                               ! INITIALIZING BNM.
2400
     Erns≠2.0/Ka
                               ! CALCULATE BINC.
2410
```

```
I * CALCULATE THE L "NAKED"
2420
        FOR Ordent=1 TO L
                                           1 * NUMR FACTORS OF J(0)/J(L). *
2430
2440
          Bnm=Bnm+Binc
                                           I INCREMENT BNM.
          IF Nflag=0 THEN
                                          ! CHECK NUMR=0 FLAG NOT SET;
2450
            Numr=Bnm-Numr
                                           ! UPDATE NUMR IF NUMR<>0.
2460
            IF Numr<>0 THEN
2470
              R1=R1+Numr
                                           ! UPDATE RL IF UPDATED NUMR<>0.
2480
2490
              Numr=1.0/Numr
                                           I PREPARE NUMR FOR NEXT EVOLUTION.
                                           I CHECK SCALING OF UPDATED RL.
2500
2510
             IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
2520
            ELSE
2530
              Nflag=1
2540
            END IF
2550
          ELSE
                                           ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
2560
            Nflag=0
                                           ! DEFERRED FOR ONE INCREMENT OF BNM.
2570
          END IF
        NEXT Ordent
2580
                                           1 * CALCULATE THE REMAINING NUMR *
2590
                                           1 * AND DENR TERMS OF J(0)/J(L). *
2600
2610
                                               NOTE WHEN NUMR-DENR THEN
2620
                                               RL=J(0)/J(L).
        WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
2630
          Bnm=Bnm+Binc
                                           I INCREMENT BNM.
2640
2650
          IF Nflag=0 THEN
                                           ! CHECK NUMR=0 FLAG NOT SET:
2660
2670
            Numr=Bnm-Numr
                                           ! UPDATE NUMR IF NUMR<>0.
2680
            IF Numr<>0 THEN
2690
              R1=R1 * Numr
                                           ! UPDATE RL IF UPDATED NUMR<>0.
                                           I PREPARE NUMR FOR NEXT EVOLUTION.
2700
              Numr=1.0/Numr
                                           I CHECK SCALING OF UPDATED RL.
2710
2720
              IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
2730
            ELSE
2740
              Nflag=1
            END IF
2750
                                           I IF NFLAG<>0 THE PREVIOUS STEPS ARE
2760
          ELSE
2770
           Nflag=0
                                           I DEFERRED FOR ONE INCREMENT OF BNM.
2780
          END IF
2790
                                           ! CHECK DENR=0 FLAG NOT SET;
          IF Oflag=0 THEN
2800
                                           ! UPDATE DENR IF DENR<>0.
2810
            Denr=Bnm-Denr
2820
            IF Denr<>0 THEN
2830
              R1=R1/Denr
                                           I UPDATE RL IF UPDATED DENR<>0.
                                           ! PREPARE DENR FOR NEXT EVOLUTION.
2840
              Denr=1.0/Denr
                                           ! CHECK SCALING OF UPDATED RL.
2850
              IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
2860
2870
            ELSE
2880
              Dflag=1
2890
            END IF
                                           ! IF DFLAG<>0 THE PREVIOUS STEPS ARE
2900
          ELSE
                                           ! DEFERRED FOR ONE INCREMENT OF BNM.
2910
            Dflag≈0
2920
          END IF
2930
                                           ļ
2940
        END WHILE
```

```
2950
2960
2970
 J(L)=J(0)/(R1*10^(Scexp))
             1 *** J(L) DETERMINED. ***
2980
2990 NEXT L
3000
3010
3020
 3030 !
3050
3060 SUBEND
3070
3090 1
3110 1
```

```
3120 1
3140 SUB Scale(R1,Scexp)
THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
3160 REM
3170 REM ONGOING CALCULATION OF RL.
3190 !
3210
3220
3230 IF R1>10-250 THEN
  R1=R1+10^(-250)
3240
3250
  Scexp≃Scexp+250
3260 END IF
3270 IF R1<10^(-250) THEN
3280
  R1=R1+10^250
3290
  Scexp=Scexp-250
3300 END IF
3310
3320
3360
               1
3370 SUBEND
3380
3400 1
3420 1
```

```
3430 I
3440
   3450 SUB Disb(Lmax, Ka, Dy(+), Y(+))
3460 REM
     3470 REM
     THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
3480 REM
      BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX. THE
3490 REM
      CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
3500 REM
     PREVIOUS ORDER.
     3510 REM
3520 1
3540
3550
3560 CALL Isb(Lmax, Ka, Y(*))
                      I CALCULATE THE ISB VALUES.
3570
3580 RAD
                      I ANGLE UNITS FOR CALC'NS.
3590 Dy(0)=(-Y(0)+SIN(Ka))/Ka
                      I THE INITIAL ISB DERIVATIVE.
3600 FOR L=1 TO Lmax
                      I THE REMAINING ISB DERIVATIVES.
3610
    Dy(L)=-((L+1)/Ka)+Y(L)+Y(L-1)
3620 NEXT L
3630
3640
  3650
3660 !
3670
  3680
3690 SUBEND
3700
3710
   3720 !
3740 !
```

```
3750 !
3770 SUB Isb(Lmax, Ka, Y(*))
THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
3790 REM
3800 REM OF ARGUMENT KA AND ORDER L THROUGH LMAX. 1. CALCULATION IS
3810 REM ACCOMPLISHED BY FORWARD RECURSION.
3820 REM
    3830 !
3850
3860
3870 RAD
                  ! ANGLE UNITS FOR CALC'NS.
3880 Y(0)=-CO5(Ka)/Ka
                  ! INITIAL ISB'S CALCULATED.
3890 Y(1)=(Y(0)-SIN(Ka))/Ka
3900 FOR L=2 TO Lmax
                  ! REMAINING ISB'S CALC'D.
  Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
3910
3920 NEXT L
3930
3940
                  ļ
3950
  3960
  ļ
3980
3990 SUBEND
4000
4020 !
4040 !
```

```
4050
4070 SUB Leg180(Lmax,Ka,P(*))
THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT -1 AND
4090 REM
    ORDER Ø THROUGH LMAX. CALCULATION IS ACCOMPLISHED BY FORWARD
4100 REM
4110 REM
    RECURSION.
    4120 REM
4130 !
4150
                  ! ARGUMENT OF THE LEG. POL'S.
4160 X=-1
4170
                  ! INITIAL LEG. POL'S. CALC'D.
4180 P(0)=1
4190 P(1)=X
4200 FOR L=2 TO Lmax
                  ! REMAINING LEG. POL'S. CALC'D.
  P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
4210
4220 NEXT L
4230
4250 1
4270
                  1
4280 SUBEND
4290
4310 !
  4320
4330 |
```

```
4340
4350
    4360 SUB Recplt(Nbs(*), Kamin, Kamax, Stp)
       REM
       THIS MODULE MAPS THE CALCULATED SCATTERING MODULUS ARRAY SM(*) ON
4390
   REM
       A SEMI-LOG PLOT.
   REM
      4410
4420
   4430
4440
4450 REAL Xgumax
                           ! THE MAXIMUM ABSCISSA VALUE IN
4460
                           ! GRAPHICS DISPLAY UNITS.
4470
4480 REAL Youmax
                           ! THE MAXIMUM ORDINATE VALUE IN
4490
                           I GRAPHICS DISPLAY UNITS.
4500
4510
4520
   4530
4540
   4550
4560 GINIT
4570 PLOTTER IS CRT, "INTERNAL"
4580 GRAPHICS ON
4590 GCLEAR
4620 !
4640
4650
4660
   Xgumax=100*MAX(1,RATIO)
4670 Ygumax=100+MAX(1,1/RATIO)
4680
4690 CSIZE 4
4700 LINE TYPE 1
4710
4720 LORG 4
                           ! ** LABELLING THE ABSCISSA AXIS **
4730 MOVE Xgumax/2,0
4740 LABEL "K+A"
4750
4760 DEG
                           ! ** LABELLING THE ORDINATE AYIS **
4770 LDIR 90
4780 LORG 6
4790 MOVE 0, Ygumax/2
4800 LABEL "NORM'D BCKSCTRG CROSS SEC'N"
4810 LDIR 0
4820
                           ! ** GRAPH CONSTRUCTION **
4830 VIEWPORT 10, Xgumax-5, 10, Ygumax-5
                          ! DEFINES THE TOTAL PLOTTING AREA
4840
                           ! IN GRAPH UNITS.
4850
4860 WINDOW Kamin Kamax ,0,1.2
4870 LINE TYPE 4
4880 GRID Stp * 10, . 1, Kamin , 0, 61, 10, . 2
```

```
4890
4900 CLIP OFF
4910 CSIZE 3
4920 LINE TYPE 1
4930
                   ! ** LABELLING THE ABSCISSA SCALE **
4940 LORG 6
4950 FOR I=Kamin TO Kamax STEP Stp*20
  MOVE I,0
4950
4970
  LABEL I
4980 NEXT I
4990
5000 LORG 8
                   ! ** LABELLING THE ORDINATE SCALE **
5010 FOR I=0 TO 1 STEP .5
5020 MOVE Kamin, I
5030 LABEL I
5040 NEXT I
5050
5060 CLIP ON
5070
5080
5100
5120
5130 FOR I=Kamin+Stp TO Kamax STEP Stp
5140
  PLOT I, Nbs((I-Kamin)/Stp)
5150 NEXT I
5160
5180 !
5200
5210 DUMP GRAPHICS #701
5220
                  - 1
5240
5250
  5260
                  1
5270 GRAPHICS OFF
5280 SUBEND
5290
5310 !
```

## APPENDIX\_D

## ACOUSTIC ARRAY EFFICIENCY PROGRAM

This program was used to analyze data and determine the calibration product  $E_r E_t$ . The output of this program was also used to construct Figures 13, 14, and 15.

The power supply mentioned in the Gain30 and Gain35 subprograms was the HP3314A function generator. All data contained in the data subprograms are repeated in Appendix E. Data units are available in Appendix E with their respective data.

```
10
    20
30
   REM
        REM
                   ACOUSTIC ARRAY EFFICIENCY
40
50
   REM
          THIS PROGRAM CALCULATES THE EFFICIENCY AND CENTERLINE GAIN OF
60
   REM
70
   REM
       AN ACOUSTIC ARRAY BASED ON THE RETURN FROM AN ACOUSTICALLY HARD
        SPHERE OF DIAMETER D ENSONIFIED BY AN INCIDENT PLANE WAVE OF
   REM
90
   REM
       FREQUENCY F.
100
   REM
130
   REM
       140
    150
160
170
   REAL Vmsr
                            I MEAN SQUARE VOLTAGE RECEIVED FROM
180
190
                            ! THE TARGET SPHERE AND MEASURED
200
                            ! AFTER PRE-AMPLIFICATION.
210
                            ! MEAN SQUARE VOLTAGE RECEIVED IN
220
   REAL Umsn
                            ! THE ABSCENCE OF A TARGET AND
230
                            I MEASURED AFTER PRE-AMP.
240
250
```

260	REAL	Vmst	! MEAN SQUARE VOLTAGE TRANSMITTED.
270 280 290 300 310 320	REAL		THE RANGE FROM THE PLANE OF THE ARRAY SPEAKER DIAPHRAGMS TO THE CENTER OF THE TARGET SPHERE AND TO THE SPECTRUM ANALYZER MICROPHONE.
330 340 350 360	REAL	Et	THE EFFICIENCY OF CONVERSION FROM TRANSMITTED ELECTRICAL POWER TO TRANSMITTED ACOUSTICAL POWER.
370 380 390	REAL	Go	THE CENTERLINE GAIN OF THE ACOUSTIC! ARRAY.
400 410	REAL	Etgo	ET+GO
420 430 440	REAL		! THE ATTENUATION COEFFICIENT, ! "ALPHA".
450	REAL		THE SPEED OF SOUND IN HUMID AIR.
460 470 480 490	REAL	Efreq · .	THE SELECTED FREQUENCY OF THE ACOUSTIC ARRAY.
500 510	REAL	•	THE WAVE NUMBER, 2*PI*EFREQ/C.
520 530	REAL	A	THE RADIUS OF THE TARGET SPHERE.
540 550 560 570	REAL		K*A, THE ARGUMENT OF THE REGULAR ! AND IRREGULAR SPHERICAL BESSEL ! FUNCTIONS AND THEIR DERIVATIVES.
580 590	REAL		THE APERATURE AREA OF THE ARRAY.
600	REAL		THE PRE-AMP'S ELECTRICAL GAIN.
610 620 630 640 650	REAL		THE VARIABLE REPRESENTING THE NORMALIZED, BACKSCATTERED CROSS SECTION (NBCS).
660 670	REAL	_	CROSS SECTIONAL AREA OF THE TARGET SPHERE.
690 700 710 720 730	REAL		SURFACE AREA OF AN IMAGINARY SPHERE CENTERED ON THE ARRAY AND HAVING A RADIUS EQUAL TO THE RANGE TO THE CENTER OF THE TARGET SPHERE.
740 750 760 770 780	REAL	Prat	THE RATIO OF RECEIVED TARGET ELECTRICAL POWER TO THE TRANSMITTED ELECTRICAL POWER.

```
! THE EFFICIENCY OF CONVERSION
790
   REAL Er
                          I FROM RECEIVED ACOUSTICAL POWER
800
                           ! TO RECEIVED ELECTRICAL POWER.
810
820
830
    840
850
860
    870
880
890
   CALL Init(Umsr, Umsn, Umst, R, Etgo, At, A, Ka, Aa, Gpre)
900
                          ! INPUT AND CALCULATE REQUIRED
                          ! PARAMETERS.
910
920
                          ! CALCULATE THE NBCS.
930
   CALL Boksotr(Ka,Nbs)
  PRINT "NBS=";Nbs
940
950
   Atgt=PI+A+A
960
970
   PRINT "ATGT=";Atgt
   Sasph=4*PI*R*R
980
990
   Prat=(Umsr-Umsn)/(Gpre+Gpre+Umst)
1000 Er=(Prat)*(Sasph*Sasph/Etgo)*(EXP(2*At*R)/(Aa*Atgt))/Nbs
1010
1020 PRINT "Er=":Er
1030 PRINT "EtGo=";Etgo
1040 PRINT "Go=";Etgo/Er
1050 PRINT "EtEr=";Er+Er
1060
1070
1090
   1100
1110
1120 PRINT CHR$(12)
1130 PRINTER IS 1
1140 PRINT
                          ! ADVISES THE USER OF THE MAIN
1150 PRINT "END"
1160 PRINT
                          ! PROGRAM'S CONCLUSION.
1170 END
1180
1200 !
1220 !
1230
```

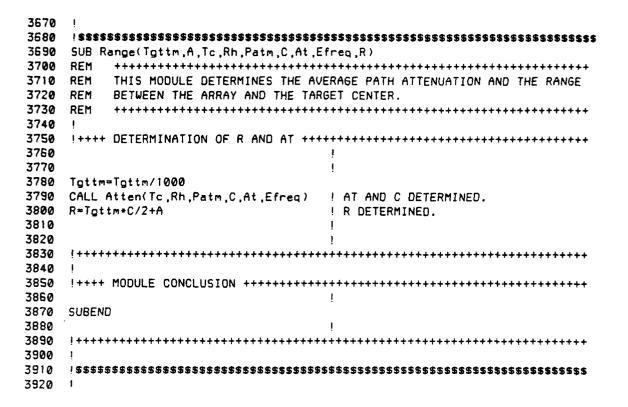
```
1250 SUB Init(Vmsr.Vmsn,Vmst,R,Etgo,At,A,Ka,Aa,Gpre)
1260 REM
         *****************
1270 REM
          THIS MODULE DETERMINES THE VALUES OF THE PARAMETERS REQUIRED FOR
1280 REM
         THE CALCULATION OF Er.
1300 !
1320
1330
1340 REAL Ch
                                   ! A CONTROL VARIABLE USED FOR
1350
                                    ! CONTROLLING SELECTION OF THE
1360
                                   ! CORRECT GAIN DATA SETS.
1370
1380 REAL Vr(60)
                                   ! THE ARRAY REPRESENTING THE
1390
                                   I MEASURED TARGET RETURN VOLTAGES.
1400
1410 REAL J.Jt.Jn.Jtn
                                   ! THE NUMBER OF ARRAY ELEMENTS MINUS
1420
                                     ONE AND A LOOP CONTROL VARIABLE.
1430
1440 REAL To
                                   ! THE AMBIENT TEMPERATURE IN DEG. C.
1450
1460 REAL Patm
                                   ! THE ATMOSPHERIC PRESSURE MEASURED
1470
                                   ! IN MILLIBARS.
1480
                                   ! THE RELATIVE HUMIDITY MEASURED IN
1490 REAL Rh
1500
                                   ! PERCENT.
1510
1520 REAL Tottm
                                   ! THE MEASURED TIME FOR THE ACOUSTIC
1530
                                   ! SIGNAL TO MAKE THE ROUND TRIP
1540
                                   ! BETWEEN THE ARRAY AND THE TARGET.
1550
1560 REAL Dia
                                   I THE MEASURED DIAMETER OF THE
1570
                                   ! TARGET SPHERE.
1580
                                   ! THE NUMBER OF SPEAKERS IN THE
1590 REAL Nuspkr
                                   ! ARRAY.
1500
1610
                                   ! THE AVERAGE DIAMETER OF AN ARRAY
1620 REAL Diaspkr
                                   ! SPEAKER.
1630
1640
1650 REAL Vt(60)
                                   ! THE ARRAY REPRESENTING THE
1660
                                   ! MEASURED TRANSMISSION VOLTAGES.
1670
                                   ! THE ARRAY REPRESENTING THE
1680 REAL Noise(60)
                                   ! MEASURED NOISE RETURN VOLTAGES.
1690
1700
1710 REAL Thoise(60)
                                   ! THE ARRAY REPRESENTING THE
                                   ! MEASURED TRANSMISSION VOLTAGES FOR
1720
                                   I NOISE DETERMINATION.
1730
1740
1750 REAL Ton
                                   ! To FOR NOISE DETERMINATION.
1760
                                   ! Rh FOR NOISE DETERMINATION.
1770 REAL Rhn
1780
```

```
1790 REAL Patmn
                                       ! Patm FOR NOISE DETERMINATION.
1800
1810 REAL Efrean
                                       ! Efreqn=Efreq.
1820
1830 REAL Cn
                                       ! C FOR NOISE DETERMINATION.
1840
1850 REAL Atn
                                       ! At FOR NOISE DETERMINATION.
1860
1870 REAL Umstn
                                       ! Vmst FOR NOISE DETERMINATION.
1880
1890 REAL Rapkr
                                       ! THE AVERAGE RADIUS OF AN ARRAY
1900
                                       ! SPEAKER.
1910
1920
1930
     1940
     1950
1960
1970
1980 PRINTER IS 1
1990 PRINT "WHICH TARGET DATA SET ";
2000 PRINT "DO YOU WISH ANALYZED?"
2010 PRINTER IS 701
2020 INPUT Sphere
2030 PRINT "TARGET NO. "ISphere
2040 PRINT
2050
2060 IF Sphere=1 THEN
2070
        Ch=3.0
        CALL Gaino(Ch, Etgo)
2080
2090
        CALL Sphere1(Vr(*), J.Tc., Patm, Rh., Tgttm, Dia, Efreq, Nuspkr, Diaspkr, Gpre)
2100
        CALL Trans30(Jt.Vt(*))
2110
        CALL Noise30(Jn, Jtn, Noise(*), Tnoise(*), Tcn, Rhn, Patmn, Efreqn)
2120 END IF
2130
2140 IF Sphere=2 THEN
2150
        Ch=3.5
2160
        CALL Gaino(Ch, Etgo)
2170
        CALL Sphere2(Vr(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,6pre)
2180
        CALL Trans35(Jt.Vt(*))
2190
        CALL Noise35(Jn, Jtn, Noise(*), Tnoise(*), Tcn, Rhn, Patmn, Efreqn)
2200 END IF
2210
2220 IF Sphere=3 THEN
2230
        Ch=3.5
2240
        CALL Gaino(Ch.Etgo)
2250
        CALL Sphere3(Vr(*), J.Tc. Patm.Rh.Tgttm.Dia.Efreq.Nuspkr.Diaspkr.Gpre)
2260
        CALL Trans35(Jt,Vt(*))
2270
        CALL Noise35(Jn,Jtn,Noise(*),Tnoise(*),Tcn,Rhn,Patmn,Efreqn)
2280 END IF
2290
```

```
2300 IF Sphere=4 THEN
       Ch=3.5
2310
2320
       CALL Gaino(Ch, Etgo)
2330
       CALL Sphere4(Vr(*), J, Tc, Patm, Rh, Tgttm, Dia, Efreq, Nuspkr, Diaspkr, Gpre)
2340
       CALL Trans35(Jt,Vt(+))
2350
       CALL Noise35(Jn, Jtn, Noise(+), Tnoise(+), Tcn, Rhn, Patmn, Efregn)
2360 END IF
2370
2380 CALL Voltsms(Ur(*),J,Umsr)
2390 PRINT "UMSR=";Umsr
2400 CALL Atten(Tcn,Rhn,Patmn,Cn,Atn,Efregn)
2410 CALL Voltsms(Tnoise(*),Jtn,Vmstn)
2420 CALL Voltsms(Noise(+), Jn, Vmsn)
2430 CALL Voltsms(Vt(*),Jt,Vmst)
2440 PRINT "VMST="; Umst
2450
                                ļ
2460 A=Dia/2
2470 PRINT "A=" ; A
2480 CALL Range(Tgttm,A,Tc,Rh,Patm,C,At,Efreq,R)
2490 PRINT "C=";C
2500 PRINT "AT="; At
2510 PRINT "ATTEN="; EXP(At*R)
2520 PRINT "R=";R
2530 Umsn=Umsn*EXP(2*R*(Atn-At))*Umst/Umstn
2540 PRINT "UMSN=":Umsn
2550 Ka=2*PI*Efreq*A/C
2560 PRINT "KA=";Ka
2570 Rspkr=Diaspkr/2
2580 Aa=Nuspkr+PI+Rspkr+Rspkr
2590 PRINT "AA="; Aa
2600
2610
2620
    2630
2540
    2650
2660
    SUBEND
2670
2690
2710 !
```

```
2720
2730
   2740
   SUB Gaino(Ch.Etgo)
2750
   REM
      2760
   REM
      THIS MODULE DETERMINES THE VALUE OF ELGO.
2770
   REM
      2780
2790
   2800
2810
                      ! ROOT MEAN SQUARE VOLTAGE MEASURED
2820
   REAL Vrms
2830
                      ! IN GAIN DETERMINATION.
2840
2850
   REAL In
                      ! MEASURED INTENSITY AT R;
2860
                      Ptgt/Atgt.
2870
2880 REAL Za
                     ! ACOUSTIC ARRAY IMPEDANCE.
2890
2900
2910
   2920
2930
   2940
2950
2960 IF Ch=3.0 THEN CALL Gain30(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
2970
  IF Ch=3.5 THEN CALL Gain35(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
2980
                     ! OBTAINING VALUES FROM APPROPRIATE
2990
                     ! DATA.
3000
3010 CALL Atten(Tc,Rh,Patm,C,At,Efreq)
                     ! CALCULATING AT.
3020 Pt=Vrms+Vrms/Za
3030
   Etgo=Ir*4*PI*R*R*EXP(At*R)/Pt
                     ! CALCULATING EtGo.
3040
3050
3060
   3070
3080
   3090
3100
   SUBEND
3110
3120
   3130
3140
   3150
```

```
3160
3170
   3180
   SUB Voltsms(Volt(+),J.Ums)
3190
       3200
   REM
       THIS MODULE DETERMINES THE MEAN SQUARE VALUE OF THE INPUT VOLTAGE
3210
   REM
       ARRAY.
3220
   REM
       3230
3240
   3250
3260
   !REAL VOLT(+)
                        I THE ARRAY REPRESENTING THE INPUT
3270
3280
                        ! VOLTAGE ARRAY.
3290
3300 REAL Volts
                        ! THE SUM OF VOLT(*)'S ELEMENTS.
3310
3320 REAL Vavo
                        ! THE AVERAGE VALUE OF VOLT(*).
3330
3340 REAL V
                        ! THE DIFFERENCE BETWEEN VOLT(I)
3350
                        ! AND VAUG.
3360
3370
   !REAL Ums
                        1 THE MEAN SQUARE VALUE OF VOLT(+).
3380
3390
3400
   3410
3420
   3430
3440
3450 Volts=0
3460 FOR I=0 TO J
3470
    Volts=Volt(I)
3480
   NEXT I
3490
   Vavq=Volts/(J+1)
3500
   FOR I=0 TO J
3510
    V=Volt(I)-Vavo
3520
    Ums=(U+V)+Ums
3530
   NEXT I
3540 Vms=Vms/(J+1)
3550
3560
3570
3580
3590
   3600
3610
   SUBEND
3620
3630
3640
3650
   3660
```



```
3930
    3940
3950 SUB Atten(Tc.Rh.Patm.Cmoist.At.Efreq)
3960
    REM
         ************************
         THIS MODULE CALCULATES THE ATTENUATION USING RELATIVE HUMIDITY
3970
    REM
3980
    REM
         IN PERCENT, ATMOSPHERIC PRESSURE IN MILLIBARS, AND TEMPERATURE
3990
    REM
         IN DEGREES CENTIGRADE.
4000
    REM
         4010
4020
    4030
4040
4050 REAL Tk
                               ! THE AMBIENT TEMPERATURE IN DEG. K.
4060
4070 REAL Es
                               ! THE SATURATION VAPOR PRESSURE.
4080
4090 REAL Pratio
                               ! THE RATIO OF WATER PRESSURE IN
4100
                               ! MILLIBARS TO ATMOSPHERIC PRESSURE
4110
                               ! IN MILLIBARS.
4120
4130 REAL Cdry
                               ! THE SPEED OF SOUND IN DRY AIR.
4140
    REAL H
4150
4160
4170 REAL Past
                               ! P ASTERISK.
4180
4190 REAL Tast
                               ! T ASTERISK.
4200
4210 REAL Fm
                               ! MAX. FRED.
4220
4230 REAL Amax
                               ! MAX EXPECTED ATTENUATION.
4240
4250 REAL Frat
                               ! EFREQ/FM
4260
4270 REAL F2
                               ! FRAT+FRAT
4280
                               ! ATTENUATION EXPONENT COMPONENTS.
4290 REAL Acl, Amol
4300
4310
4320
    4330
    1
```

```
4350
4360
4370 Tk=Tc+273.15
                            ! CONVERSION OF TEMP IN CELSIUS TO
                            ! TEMP IN KELVIN.
4380
4390
                            ! CONVERSION OF RH IN PCT. TO PRATIO.
4400 Rh=Rh/100
4410 Es=10^(9.4-2353/Tk)
4420 Pratio=Es*Rh/(Patm-Es*(1+Rh))
4430
4440 Cdry=20.05*SQR(Tk)
                           ! CALCULATION OF THE SPEED OF SOUND.
4450 Cmoist=Cdry+(1+.14*Pratio)
4460
4470 H=Pratio+100
4480 Past=Patm/1014
   Tast=(1.8*Tc+492)/519
4490
4500 Fm=(10+6500+H+44400+H+H)*Past/(Tast^.8)
4510
4520 Amax=.0078*Fm*(Tast^(-2.5))*EXP(7.77*(1-1/Tast))
4530
4540 Frat=Efreq/Fm
4550 F2=Frat+Frat
4560
4570 Amol=Amax*SQR(.18*.18*F2+(2*F2/(1+F2))^2)/304.8
4580 Acl=1.74*10^(-10)*Efreq*Efreq
4590 At=(Amol+Acl)*L06(10)/10
4600
4510
4620
    4630
    4640
4650
4660
   SUBEND
4670
4680
    4690
   4700
4710
```

```
4720 |
4740 SUB Bcksctr(Ka,Nbs)
4760 REM
         THIS MODULE CALCULATES THE BACKSCATTERED CROSS SECTION NORMALIZED
4770 REM
          BY THE CROSS SECTIONAL AREA OF THE SPHERE.
4780 REM
          4790 !
4810
4820
4830 REAL L
                                  ! ORDER OF THE FUNCTION CALCULATED
4840
                                  ! IN A SUBROUTINE. IT IS USED
4850
                                  ! AS A LOOP INDEX AND TO DESIGNATE
4860
                                  ! THE ARRAY ELEMENT FOR THE FUNCTION
4870
                                    OF CORRESPONDING ORDER.
4880
4890 REAL Lmax
                                  ! MAX. SIGNIFICANT ORDER OF THE SUM
4900
                                    DETERMINING THE NORMALIZED
4910
                                    BACKSCATTERING CROSS SECTION
4920
                                  ! (NBCS).
4930
4940 REAL J(51)
                                  ! ARRAY FOR THE REGULAR SPHERICAL
4950
                                  ! BESSEL FUNCTIONS (RSB) OF
4960
                                  ! ARGUMENT KA AND ORDER Ø THROUGH
4970
                                  ! LMAX.
4980
                                  ! ARRAY FOR THE DERIVATIVES OF THE
4990 REAL Di(51)
5000
                                  ! RSB (THE DRSB).
5010
5020 REAL Y(51)
                                  ! ARRAY FOR THE IRREGULAR SPHERICAL
5030
                                    BESSEL FUNCTIONS (ISB) OF
5040
                                    ARGUMENT KA AND ORDER Ø THROUGH
5050
5060
5070 REAL Dy(51)
                                  ! ARRAY FOR THE DERIVATIVES OF THE
5080
                                   ISB (THE DISB).
5090
5100 REAL P(51)
                                  ! ARRAY FOR THE LEGENDRE
5110
                                  ! POLYNOMIALS OF ARGUMENT COS(180).
5120
5130 REAL Fac
                                  I FACTOR COMMON TO RSM AND ISM.
5140
5150 REAL Rsm
                                  ! THE REAL COMPONENT OF THE
5160
                                  ! SCATTERING MODULUS, SM.
5170
5180 REAL Ism
                                  ! THE IMAGINARY COMPONENT OF THE
5190
                                  ! SM.
5200
```

```
! ++++ NOTE ++++
5210
                             ! THE ARRAY DIMENSIONS (30+21) NEED
5220
                             ! TO BE INCREASED TO ACCOMODATE
5230
                             ! KA>30.
5240
5250
5260
    5270
5280
5290
    5300
5310 Lmax=INT(Ka+21)
5320
5330 'CALL Drsb(Lmax, Ka, Dj(+), J(+))
                             ! CALCULATE THE DERIVATIVES OF THE
5340
                             ! REGULAR SPHERICAL BESSEL FUNCTIONS
5350
                             ! OF ORDER Ø THROUGH LMAX.
5360
    CALL Disb(Lmax, Ka, Dy(*), Y(*))
                             ! CALCULATE THE DERIVATIVES OF THE
5370
                             ! IRREGULAR SPHERICAL BESSEL FUNC'NS
5380
                             ! OF ORDER 0 THROUGH LMAX.
5390
5400 CALL Leg180(Lmax, Ka, P(*))
                             ! CALCULATE THE ARRAY P(*).
5410
                             ! CALCULATING THE REAL AND IMAGINARY
5420 Rsm=0
                             ! COMPONENTS OF THE SM.
5430 Ism=0
5440 FOR L=0 TO Lmax
5450
     Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
5460
     Rsm=Rsm+(Fac+Dy(L))
5470
     Ism=Ism+(Fac*Dj(L))
5480 NEXT L
5490
5500 Nbs=4*(Rsm*Rsm+Ism*Ism)/(Ka*Ka)
                             ! CALCULATING THE NBCS.
5510
5520
5530
    5540
5550
    5550
5570
    SUBEND
5580
    5590
5600
5610
   5620 !
```

```
5630 !
5650 SUB Drsb(Lmax,Ka,Dj(+),J(+))
5660 REM
      THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
5670 REM
      BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS Ø THROUGH LMAX.
5680
   REM
5690
   REM
      THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
5700
   REM
      AND PREVIOUS ORDER.
5710
   REM
      5720
5730
   5740
5750
                       ! CALCULATE THE RSB VALUES J(+).
5760 CALL Rsb(Lmax, Ka, J(+))
5770
                       ! ANGLE UNITS FOR DRSB CALC'NS.
5780 RAD
5790 D_{j}(0)=(-J(0)+COS(Ka))/Ka
                       ! CALCULATE THE INITIAL DRSB DJ(0).
                       ! CALCULATE THE REMAINING DRSB.
5800 FOR L=1 TO Lmax
    D_{j}(L)=-((L+1)/Ka)*J(L)+J(L-1)
5810
5820 NEXT L
5830
5840
   5850
5860
   5870
5880
5890
   SUBEND
5900
   <u>|</u>
5910
5920
   5930
5940 !
```

```
5950 !
5970 SUB Rsb(Lmax, Ka, J(*))
5980 REM
        5990
        THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
    REM
        (RSB) OF ARGUMENT KA AND ORDERS @ THROUGH LMAX.
6000
    REM
6010
    REM
        CALCULATION USES THE CONTINUED FRACTION APPROACH.
5020
    REM
        6030
    5040
6050
6060
6070 REAL R1
                              THE EVOLVING RATIO OF J(0)/J(L).
5080
6090 REAL Numr
                             ! THE EVOLVING NUMERATOR FACTOR USED
6100
                             ! IN CALCULATION OF RL.
6110
6120 REAL Denr
                             ! THE EVOLVING DENOMINATOR FACTOR
6130
                             ! USED IN CALCULATION OF RL.
6140
6150 REAL Scexp
                             ! EVOLVING SCALING EXPONENT FOR VERY
                             ! LARGE OR VERY SMALL VALUES OF RL.
6160
6170
6180 REAL Nflag
                             ! 0 FLAG FOR NUMR.
6190
6200 REAL Dflag
                             ! 0 FLAG FOR DENR.
6210
6220 REAL Bnm
                             ! EVOLVING TERM USED IN THE
6230
                             ! CALCULATION OF NUMR AND DENR.
6240
6250 REAL Binc
                             ! INCREMENT USED IN THE EVOLUTION
6260
                             ! OF BNM.
6270
6280
6300
6310
   6320
6330
6340 RAD
                             ! ANGLE UNITS FOR CALC'NS.
6350
6360
                             ! *** CALCULATE THE INITIAL ***
6370 J(0)=(SIN(Ka))/Ka
                             ! *** RSB. J(0).
6380
```

```
6390
                                            ! *** CALCULATING THE RATIO ***
6400 FOR L=1 TO Lmax
                                            ! *** J(0)/J(L).
        R1=1.0
6410
                                            ! INITIALIZING RL.
        Numr=0.
6420
                                            ! INITIALIZING NUMR.
6430
        Denr=0.
                                            ! INITIALIZING DENR.
6440
        Scexp=0.
                                             INITIALIZING SCEXP.
6450
        Nflag=0.
                                              INITIALIZING NFLAG.
6460
        Dflag=0.
                                              INITIALIZING DFLAG.
6470
        Bnm=1.0/Ka
                                             INITIALIZING BNM.
6480
        Binc=2.0/Ka
                                            ! CALCULATE BING.
5490
6500
                                           ! * CALCULATE THE L "NAKED"
6510
        FOR Ordent=1 TO L
                                           ! * NUMR FACTORS OF J(0)/J(L). *
6520
          Bnm=Bnm+Binc
                                           I INCREMENT BNM.
6530
          IF Nflag=0 THEN
                                           ! CHECK NUMR=0 FLAG NOT SET;
6540
            Numr=Bnm-Numr
                                            ! UPDATE NUMR IF NUMR<>0.
            IF Numr<>0 THEN
6550
6560
              R1=R1*Numr
                                           I UPDATE RL IF UPDATED NUMR <>0.
6570
              Numr=1.0/Numr
                                           I PREPARE NUMR FOR NEXT EVOLUTION.
6580
                                            ! CHECK SCALING OF UPDATED RL.
6590
              IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
6600
            ELSE
6610
              Nflag=1
            END IF
6620
                                           I IF NFLAG<>0 THE PREVIOUS STEPS ARE
6630
          ELSE
6640
            Nflag=0
                                            ! DEFERRED FOR ONE INCREMENT OF BNM.
6650
          END IF
6660
        NEXT Ordent
6670
                                             * CALCULATE THE REMAINING NUMR *
6680
                                             * AND DENR TERMS OF J(0)/J(L). *
6690
                                                NOTE WHEN NUMR=DENR THEN
6700
                                                RL=J(0)/J(L).
        WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
6710
          Bnm=Bnm+Binc
6720
                                            ! INCREMENT BNM.
6730
          IF Nflag≈0 THEN
6740
                                           I CHECK NUMR=0 FLAG NOT SET;
6750
            Numr=Bnm-Numr
                                            ! UPDATE NUMR IF NUMR<>0.
            IF Numr<>0 THEN
6760
                                           ! UPDATE RL IF UPDATED NUMR<>0.
6770
              R1=R1 * Numr
6780
                                            ! PREPARE NUMR FOR NEXT EVOLUTION.
              Numr=1.0/Numr
6790
                                            ! CHECK SCALING OF UPDATED RL.
6800
              IF ABS(R1)>10^250 THEN CALL Scale(R1,Scexp)
6810
            ELSE
6820
              Nflag=1
6830
            END IF
6840
          ELSE
                                           ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
                                            ! DEFERRED FOR ONE INCREMENT OF BNM.
6850
            Nflag=0
6860
          END IF
6870
          IF Oflag=0 THEN
                                           ! CHECK DENR=0 FLAG NOT SET;
6889
            Denr=Bnm-Denr
                                           ! UPDATE DENR IF DENR<>0.
6890
5900
            IF Denr<>0 THEN
6910
              R1=R1/Denr
                                           ! UPDATE RL IF UPDATED DENR<>0.
                                            ! PREPARE DENR FOR NEXT EVOLUTION.
6920
              Denr=1.0/Denr
```

```
! CHECK SCALING OF UPDATED RL.
6930
       IF ABS(R1)>10^250 THEN CALL Scale(R1.Scexp)
6940
6950
      ELSE
6960
       Dflag=1
6970
      END IF
                       ! IF DFLAG<>0 THE PREVIOUS STEPS ARE
6980
     ELSE
                       ! DEFERRED FOR ONE INCREMENT OF BNM.
6990
      Dflag=0
     END IF
7000
7010
    END WHILE
7020
7030
7040
7050
    J(L)=J(0)/(R1+10^{(Scexp)})
                       ! *** J(L) DETERMINED. ***
7060
7070 NEXT L
7080
7090
7110 !
7130
7140 SUBEND
7150
7170
7190 !
```

```
7200 !
  7210
7220 SUB Scale(R1,Scexp)
7240 REM THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
7250 REM ONGOING CALCULATION OF RL.
7279
7290
7300
7310 IF R1>10^250 THEN
   R1=R1+10^(-250)
7320
7330
   Scexp=Scexp+250
7340 END IF
7350 IF R1<10^(-250) THEN
  R1=R1+10^250
7360
7370
  Scexp=Scexp-250
7380 END IF
7390
7400
7420 !
7430
  7440
7450
  SUBEND
7460
  7470
7480
7490
  7500
```

```
7510 !
7530 SUB Disb(Lmax, Ka, Dy(*), Y(*))
7540 REM
      7550 REM
      THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
7560 REM
      BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS @ THROUGH LMAX. THE
7570 REM
      CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
7580
   REM
      PREVIOUS ORDER.
7590
   REM
      ****
7600
7610
   7620
7630
7640 CALL Isb(Lmax, Ka, Y(*))
                      ! CALCULATE THE ISB VALUES.
7650
7660 RAD
                      ! ANGLE UNITS FOR CALC'NS.
7670 Dy(0)=(-Y(0)+SIN(Ka))/Ka
                      ! THE INITIAL ISB DERIVATIVE.
7680 FOR L=1 TO Lmax
                      ! THE REMAINING ISB DERIVATIVES.
7690
   Dy(L)=-((L+1)/Ka)*Y(L)+Y(L-1)
7700 NEXT L
7710
7720
7730
   7740
7750
   7760
   SUBEND
7770
7780
7790
   7800
7810
  7820 1
```

```
7830 !
7840
  7850 SUB Isb(Lmax, Ka, Y(*))
7860 REM
     7870 REM
     THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
7880 REM
     OF ARGUMENT KA AND ORDER & THROUGH LMAX. THE CALCULATION IS
7890 REM
     ACCOMPLISHED BY FORWARD RECURSION.
7900 REM
     7910
7930
7940
7950 RAD
                   ! ANGLE UNITS FOR CALC'NS.
7960 Y(0)=-COS(Ka)/Ka
                   ! INITIAL ISB'S CALCULATED.
7970 Y(1)=(Y(0)-SIN(Ka))/Ka
7980 FOR L=2 TO Lmax
                   ! REMAINING ISB'S CALC'D.
7990
   Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
8000 NEXT L
8010
8020
8030
  8040 !
8060
8070 SUBEND
8080
8100 !
8120 !
```

```
8130 !
8150 SUB Leg180(Lmax.Ka.P(+))
THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT -1 AND
8170 REM
    ORDER Ø THROUGH LMAX. CALCULATION IS ACCOMPLISHED BY FORWARD
8180 REM
8190 REM
     RECURSION.
8200 REM
    8210
  ŀ
8230
                  ! ARGUMENT OF THE LEG. POL'S.
8240 X=-1
8250
                  ! INITIAL LEG. POL'S. CALC'D.
8260 P(0)=1
8270 P(1)=X
8280 FOR L=2 TO Lmax
                  ! REMAINING LEG. POL'S. CALC'D.
   P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
8290
8300 NEXT L
8310
8330
  8340
8350
8360 SUBEND
8370
  8380
8390
  8400
8410 !
```

```
8420 !
8430
  8440
  SUB Gain30(Vrms, Tc, Rh, R, Patm, Ir, Efreq, Za)
8450 REM
     8460 REM
     THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF EtGo WITH
8470
  REM
     3.0V INPUT FROM THE POWER SUPPLY.
8480
  REM
    8490
8500
  8510
8520
8530 RESTORE 630
8540 READ Vrms, Tc, Rh, R, Patm, Ir, Efreq, Za
     DATA .426,21.0,50.0,5.30,1021.5,2.951E-3,5000,15.8
8560
8570
8580
  8590
8600
  8610
8620
  SUBEND
8630
8640
  8650
8560
  8670 !
```

```
8680
  8690
8700 SUB Gain35(Vrms, Tc, Rh, R, Patm, Ir, Efreq, Za)
8710 REM
    THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF EtGo WITH
8720 REM
    3.5V INPUT FROM THE POWER SUPPLY.
8730 REM
8740
  REM
8750
8770
2780
8790 RESTORE 635
8800 READ Vrms.Tc,Rh,R,Patm,Ir,Efreq,Za
8810 G35: DATA .523,21.0,50.0,5.30,1021.5,4.467E-3,5000,15.8
8820
8830
8840
  8850
8860
  8870
8880
  SUBEND
8890
8910 !
8930 !
```

```
8940 !
   8950
8960 SUB Sphere!(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
       8970
   REM
       THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
8980
   REM
8990
   REM
       TARGET NO. 1 EMPLOYED.
9000
       REM
9010
   1
   9020
9030
9040
9050 J=39
9060 RESTORE S1
9070 FOR I=0 TO J
9080
    READ Dt(I)
9090 NEXT I
9100 READ To, Patm, Rh, Tgttm, Dia, Efreq, Nuspkr, Diaspkr, Gpre
9110 S1: DATA 9.54,9.4,8.32,6.42,3.88,.96,-2.07,-4.9,-7.24,-8.87
      DATA -9.64,-9.45,-8.32,-6.41,-3.83,-.89,2.14,4.94,7.26,8.88
9120
9130
      DATA 9.6,9.38,8.22,6.26,3.67,.73,-2.3,-5.11,-7.4,-8.96
9140
      DATA -9.64,-9.39,-8.2,-6.21,-3.61,-.65,2.36,5.12,7.38,8.91
      DATA 20.2,1016.8,52.8,29.91,.2546,5000,19,.0762,11094
9150
9160
9170
   9180
9190
   9200
9210
9220
   SUBEND
9230
9240
9250
9270 !
```

```
9280
   9290
9300 SUB Sphere2(Ot(*), J, Tc, Patm, Rh, Tgttm, Dia, Efreq, Nuspkr, Diaspkr, Gpre)
9310
       THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
9320
   REM
9330
   REM
       TARGET NO. 2 EMPLOYED.
9340
   REM
       9350
9360
   9370
9380
9390 J=39
9400 RESTORE S2
9410 FOR I=0 TO J
9420
    READ Dt(I)
9430 NEXT I
9440 READ To, Patm, Rh, Tgttm, Dia, Efreq, Nuspkr, Diaspkr, Gpre
9450 S2: DATA 4.72,4.66,4.16,3.23,2.01,.560,-.940,-2.35,-3.52,-4.36
9460
     DATA -4.77,-4.70,-4.18,-3.25,-1.99,-.560,.960,2.34,3.51,4.34
9470
     DATA 4.74,4.66,4.13,3.19,1.95,.500,-1.00,-2.40,-3.57,-4.38
     DATA -4.78,-4.68,-4.15,-3.20,-1.92,-.480,1.02,2.39,3.54,4.34
9480
9490
     DATA 21.0,1009.4,49.8,30.55,.1013,5000,19,.0762,11094
9500
9510
9520
   9530
9540
   9550
9560
   SUBEND
9570
9580
   9590
9600
   9610 !
```

```
9620 !
9640 SUB Sphere3(Dt(*), J.Tc.Patm.Rh.Tgttm.Dia,Efreq.Nuspkr,Diaspkr,Gpre)
9650 REM
9660 REM
       THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
9670
   REM
       TARGET NO. 3 EMPLOYED.
       9680
   REM
9690
9700
   9710
9720
9730 J=39
9740 RESTORE S3
9750 FOR I=0 TO J
9760
     READ Dt(I)
9770 NEXT I
9780 READ To ,Patm ,Rh ,Tgttm ,Dia ,Efreq ,Nuspkr ,Diaspkr ,Gpre
9790 S3: DATA 3.61,3.40,2.86,2.04,1.01,-.120,-1.25,-2.25,~3.04,-3.54
      DATA -3.68.-3.46.-2.91.-2.07,-1.04..110,1.25,2.24,3.00,3.48
9800
      DATA 3.63,3.39,2.84,2.02,.970,-.180,-1.30,-2.31,-3.08,-3.56
9810
9820
      DATA -3.69,-3.46,-2.89,-2.04,-.990,.160,1.29,2.26,3.05,3.50
9830
      DATA 21.3,1009.6,49.8,30.59,.07634,5000,19,.0762,11094
9840
9850
   <u>|</u>
9860
9870
   9880
9890
9900
   SUBEND
9910
9920
9930
9940
   9950 !
```

```
9960
   9970
9980 SUB Sphere4(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
9990 REM
      THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
10000 REM
10010 REM
      TARGET NO. 4 EMPLOYED.
      10020 REM
10030 !
10050
10060
10070 J=39
10080 RESTORE S4
10090 FOR I=0 TO J
10100
    READ Dt(I)
10110 NEXT I
10120 READ To,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre
10130 S4: DATA 2.28,2.10,1.68,1.10,.420,-.320,-1.03,-1.64,-2.07,-2.31
10140
     DATA -2.34,-2.12,-1.71,-1.12,-.440,.320,1.02,1.62,2.06,2.28
     DATA 2.29,2.10,1.68,1.09,.400,-.340,-1.04,-1.66,-2.10,-2.32
10150
     DATA -2.35,-2.12,-1.70,-1.11,-.400,.340,1.04,1.64,2.08,2.28
10160
     DATA 21.4,1009.9,49.6,30.64,.06240,5000,19,.0762,11094
10170
10180
10190
10210 !
10230
10240 SUBEND
10250
10290 !
```

```
10300 !
10320 SUB Trans30(J.Dt(*))
10330 REM
     10340 REM
     THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10350 REM
     3.0V APPLIED.
10360 REM
     10370 !
10390
10400
10410 J=39
10420 RESTORE T30
10430 FOR I=0 TO J
10440
    READ Dt(I)
10450 NEXT I
10460 T30: DATA .690,.680,.590,.440,.320,.160,-.020,-.230,-.410,-.550
     DATA -.610,-.610,-.520,-.360,-.240,-.080,.110,.320,.490,.630
10470
     DATA .690,.680,.590,.430,.320,.150,-.040,-.240,-.420,-.560
10480
     DATA -.620,-.600,-.510,-.380,-.240,-.070,.120,.330,.500,.640
10490
10500
10510
10530
10550
10560 SUBEND
10570
10590 !
10510 !
```

```
10620 !
10640 SUB Trans35(J.Dt(*))
      10650 REM
      THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10660 REM
10670 REM
      3.5V APPLIED.
      10680 REM
10690 !
10710
10720
10730 J=39
10740 RESTORE T35
10750 FOR I=0 TO J
    READ Dt(I)
10750
10770 NEXT I
10780 T35: DATA .830,.770,.640,.490,.290,.080,-.180,-.400,-.610,-.730
      DATA -.770, -.720, -.580, -.430, -.240, -.020, .240, .460, .660, .780
10790
      DATA .830,.770,.630,.470,.290,.060,-.190,-.420,-.600,-.720
10800
10810
      DATA -.770,-.710,-.570,-.420,-.230,-.010,.240,.480,.670,.790
10820
10830
10850 !
10870
10880 SUBEND
10890
10930
```

```
10940 !
10960 SUB Noise30(J.Jt.Dt(*),Tdt(*),Tcn,Rhn,Patmn,Efregn)
10970 REM
        10980 REM
        THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10990 REM
       3.0V APPLIED.
11000 REM
        11010 !
11030
11040
11050 J=59
11060 RESTORE N30
11070 FOR I=0 TO J
11080
     READ Dt(I)
11090 NEXT I
11100 Jt=39
11110 RESTORE Tn30
11120 FOR I=0 TO Jt
11130
     READ Tdt(I)
11140 NEXT I
11150 READ Ton, Rhn, Patmn, Efreqn
11160 N30:
       DATA 0,0,-.02,-.03,-.04,-.06,-.08,-.1,-.11,-.12
11170
       DATA -.12,-.11,-.1,-.08,-.07,-.05,-.04,-.01,0,0
       DATA 0,0,-.02,-.03,-.05,-.07,-.08,-.1,-.11,-.12
11180
       DATA -.12,-.12,-.11,-.08,-.07,-.04,-.03,-.02,0
11190
11200
       DATA .01,0,0,-.03,-.04,-.07,-.09,-.11,-.12,-.12
       DATA -.12,-.12,-.11,-.08,-.07,-.04,-.03,0,0,.01
11210
11220 Tn30: DATA .69,.68,.59,.44,.32,.16,-.02,-.23,-.41,-.55
11230
       DATA -.61,-.61,-.52,-.36,-.24,-.08,.11,.32,.49,.63
11240
       DATA .69,.68,.59,.43,.32,.15,-.04,-.24,-.42,-.56
       DATA -.62,-.6,-.51,-.38,-.24,-.07,.12,.33,.5,.64
11250
11260
       DATA 20.8,51.7,1016.7,5000
11270
11280
11320
                           1
11330 SUBEND
11340
11380 !
```

```
11390 !
11410 SUB Noise35(J,Jt,Dt(+),Tdt(+),Tcn,Rhn,Patmn,Efreqn)
       11420 REM
11430 REM
       THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
11440 REM
       3.5V APPLIED.
11450 REM
       11460 !
11480
11490
11500 J=39
11510 RESTORE N35
11520 FOR I=0 TO J
     READ Dt(I)
11530
11540 NEXT I
11550 Jt=39
11560 RESTORE Th35
11570 FOR I=0 TO Jt
     READ Tdt(I)
11580
11590 NEXT I
11600 READ Ton, Rhn, Patmn, Efreqn
11610 N35: DATA -.06,-.08,-.08,-.08,-.07,-.06,-.04,-.02,0,.02
       DATA .05,.06,.06,.06,.04,.03,.02,0,-.03,-.05
11520
       DATA -.07,-.08,-.08,-.08,-.06,-.04,-.02,.01,.03
11630
11540
       DATA .04..07..06..07..05..04..01,-.01,-.04,-.06
11650 Tn35: DATA .74,.83,.82,.72,.53,.38,.18,-.06,-.31,-.53
       DATA -.68,-.77,-.75,-.64,-.46,-.3,-.1,.14,.4,.6
11660
11670
       DATA .75,.83,.81,.7,.5,.36,.16,-.08,-.33,-.54
       DATA -.69,-.76,-.74,-.63,-.46,-.28,-.08,.16,.41,.62
11680
       DATA 21.3,51.3,1016.6,5000
11690
11700
11710
11750
11760 SUBEND
11770
11810 !
```

## APPENDIX E

## ACOUSTIC ARRAY CALIBRATION DATA

This appendix contains data used to determine the calibration product  $E_{\mu}E_{\mu}$ . Data units are contained in square brackets.

For listings of the same type of data the units and uncertainty are explicitly given for the first datum. Remaining data in the listing have the same units and uncertainty as the first datum.

## 1. TWO WAY PROPAGATION PATH DATA

Pre-amplifier gain  $G_{\rm e}$  and array impedance Z were previously determined by Moxcey.  $G_{\rm e}$  was determined to be 11,094. Z was determined to be 15.8 [O]. [Ref. 8]

The remaining data are presented in the following tables.

 $V_{\text{supply}}$  is the voltage supplied by the HP3314A function generator to the pre-amplifier (see Figure 8).

Table 5a: Two Way Propagation Path Data for Sphere One, Collected 30 November 1987; Run #2: Data Re-measured Because First Run  $V_r$  (with 3.5 [V] Supply) Was Clipped.

```
Diameter = 2a \approx .2546 \pm 0.0006 [m]

T_c = 20.2 \pm 0.1 [° c]

Rh = 52.8 \pm 0.1 [%]

P = 1016.8 \pm 0.2 [mb]

No. pulses = 20

V_{\text{supply}} = 3.0 [V]

t_r = 30.31 [mS] - 400 [\muS] = 29.91 \pm 0.1 [mS]
```

V <sub>t</sub>	t	$V_{\mathbf{t}}$
690 ± 5 [mV]	2.45	690
680	2.46	680
590	2.47	590
440	2.48	430
320	2.49	320
160	2.50	150
- 20	2.51	- 40
-230	2.52	-240
-410	2.53	-420
-550	2.54	-560
-610	2.55	-620
-610	2.56	-600
-520	2.57	-510
-360	2.58	-380
-240	2.59	-240
- 80	2.60	- 70
110	2.61	120
320	2.62	330
490	2.63	500
630	2.64	640
	690 ± 5 [mV] 680 590 440 320 160 - 20 -230 -410 -550 -610 -610 -520 -360 -240 - 80 110 320 490	690 ± 5 [mV] 2.45 680 2.46 590 2.47 440 2.48 320 2.49 160 2.50 - 20 2.51 -230 2.52 -410 2.53 -550 2.54 -610 2.55 -610 2.55 -610 2.56 -520 2.57 -360 2.58 -240 2.59 - 80 2.60 110 2.61 320 2.62 490 2.63

Table 5b: Two Way Propagation Path Data for Sphere One Continued, Collected 30 November 1987; Run \*2: Data Re-measured Because First Run  $V_r$  (with 3.5 [V] Supply) Was Clipped.

t	V <sub>r</sub>	t	V <sub>r</sub>
33.68 ±0.01[mS]	9.54 ± 0.03 [V]	33.88	9.60
33.69	9.40	33.89	9.38
33.70	8.32	33.90	8.22
33.71	6.42	33.91	6.26
33.72	3.88	33.92	3.67
33.73	0.960	33.93	0.730
33.74	-2.07	33.94	-2.30
33.75	-4.90	33.95	-5.11
33.76	-7.24	33.96	-7.40
33.77	-8.87	33.97	-8.96
33.78	-9.64	33.98	-9.64
33.79	-9.45	33.99	-9.39
33.80	-8.32	34.00	-8.20
33.81	-6.41	34.01	-6.21
33.82	<b>-</b> 3.83	34.02	-3.61
33.83	-0.890	34.03	-0.650
33.84	2.14	34.04	2.36
33.85	4.94	34.05	5.12
33.86	7.26	34.06	7.38
33.87	8.88	34.07	8.91

Table 6a: Two Way Propagation Path Data for Sphere Two, Collected 28 November 1987.

Diameter =  $2a = 10.13 \pm 0.01$  [cm]

2.44

2.45

```
T_c = 21.0 \pm 0.1 [^{\circ} c]
Rh = 49.8 \pm 0.1 [\%]
P = 1009.4 \pm 0.2 \text{ [mb]}
No. pulses = 20
V_{\text{supply}} = 3.5 \text{ [V]}
t_r = 30.95 \text{ [mS]} - 400 \text{ [}\mu\text{S]} = 30.55 \pm 0.1 \text{ [mS]}
t
                    ٧<sub>t</sub>
                                                  t
                                                                 ٧Ł
                     830 \pm 5 [mV]
2.26 \pm 0.01 [mS]
                                                  2.46
                                                                  830
2.27
                     770
                                                  2.47
                                                                  770
2.28
                     640
                                                  2.48
                                                                  630
2.29
                     490
                                                  2.49
                                                                  470
2.30
                     290
                                                  2.50
                                                                  290
2.31
                      80
                                                  2.51
                                                                   60
2.32
                    -180
                                                  2.52
                                                                 -190
2.33
                    -400
                                                  2.53
                                                                 -420
2.34
                    -610
                                                  2.54
                                                                 -600
2.35
                    -730
                                                  2.55
                                                                 -720
2.36
                    -770
                                                  2.56
                                                                 ~770
2.37
                    -720
                                                  2.57
                                                                 -710
2.38
                    -580
                                                  2.58
                                                                 -570
2.39
                    -430
                                                  2.59
                                                                 -420
2.40
                    -240
                                                  2.60
                                                                 -230
2.41
                    - 20
                                                  2.61
                                                                 - 10
2.42
                     240
                                                                  240
                                                  2.62
2.43
                     460
                                                  2.63
                                                                  480
```

2.64

2.65

670

790

660

780

Table 6b: Two Way Propagation Path Data for Sphere Two Continued, Collected 28 November 1987.

t	V <sub>r</sub>	t	V <sub>r</sub>
34.13 ±0.01[mS]	4.72 ± 0.01 [V]	34.33	4.74
34.14	4.66	34.34	4.66
34.15	4.16	34.35	4.13
34.16	3.23	34.36	3.19
34.17	2.01	34.37	1.95
34.18	0.560	34.38	0.500
34.19	-0.940	34.39	-1.00
34.20	-2.35	34.40	-2.40
34.21	-3.52	34.41	-3.57
34.22	-4.36	34.42	-4.38
34.23	-4.77	34.43	-4.78
34.24	-4.70	34.44	-4.68
34.25	-4.18	34.45	-4.15
34.26	-3.25	34.46	-3.20
34.27	-1.99	34.47	-1.92
34.28	-0.560	34.48	-0.480
34.29	0.960	34.49	1.02
34.30	2.34	34.50	2.39
34.31	3.51	34.51	3.54
34.32	4.34	34.52	4.34

Table 7: Two Way Propagation Path Data for Sphere Three, Collected 28 November 1987.

Diameter =  $2a = 7.634 \pm 0.003$  [cm]  $T_c = 21.3 \pm 0.1$  [° c]  $Rh = 49.8 \pm 0.1$  [%]  $P = 1009.6 \pm 0.2$  [mb] No. pulses = 20  $V_{supply} = 3.5$  [V]  $t_r = 30.99$  [mS] - 400 [ $\mu$ S] =  $30.59 \pm 0.1$  [mS]

 $\mathbf{V_t}$  data is the same as sphere two's (see Table 6a).

t	V <sub>r</sub>	t	V <sub>r</sub>
33.98 ±0.01[mS]	3.61 ± 0.03 [V]	34.18	3.63
33.99	3.40	34.19	3.39
34.00	2.86	34.20	2.84
34.01	2.04	34.21	2.02
34.02	1.01	34.22	0.970
34.03	-0.120	34.23	-0.180
34.04	-1.25	34.24	-1.30
34.05	-2.25	34.25	-2.31
34.06	-3.04	34.26	-3.08
34.07	-3.54	34.27	-3.56
34.08	-3.68	34.28	-3.69
34.09	-3.46	34.29	-3.46
34.10	-2.91	34.30	-2.89
34.11	-2.07	34.31	-2.04
34.12	-1.04	34.32	-0.990
34.13	0.110	34.33	0.160
34.14	1.25	34.34	1.29
34.15	2.24	34.35	2.26
34.16	3.00	34.36	3.05
34.17	3.48	34.37	3.50

Table 8: Two Way Propagation Path Data for Sphere Four, Collected 28 November 1987.

Diameter =  $2a = 6.240 \pm 0.008$  [cm]  $T_c = 21.4 \pm 0.1$  [° c]  $Rh = 49.6 \pm 0.1$  [%]  $P = 1009.9 \pm 0.2$  [mb] No. pulses = 20  $V_{supply} = 3.5$  [V]  $t_r = 31.04$  [mS] - 400 [ $\mu$ S] =  $30.64 \pm 0.1$  [mS]

 $\mathbf{V_t}$  data is the same as sphere two's (see Table 6a).

t	V <sub>r</sub>	t	٧
33.83 ±0.01[mS]	2.28 ± 0.03 [V]	34.03	2.29
33.84	2.10	34.04	2.10
33.85	1.68	34.05	1.68
33.86	1.10	34.06	1.09
33.87	0.420	34.07	0.400
33.88	-0.320	34.08	-0.340
33.89	-1.03	34.09	-1.04
33.90	-1.64	34.10	-1.66
33.91	-2.07	34.11	-2.10
33.92	-2.31	34.12	-2.32
33.93	-2.34	34.13	-2.35
33.94	-3.12	34.14	-2.12
33.95	-1.71	34.15	-1.70
33.96	-1.12	34.16	-1.11
33.97	-0.440	34.17	-0.400
33.98	0.320	34.18	0.340
33.99	1.02	34.19	1.04
34.00	1.62	34.20	1.64
34.01	2.06	34.21	2.08
34.02	2.28	34.22	2.28

Table 9: Noise Data for 3.0 [V] Supply Voltage; Collected 30 November 1987.

$T_{-} = 20.8 \pm 0.1$ [°	c] Rh = 51.7 ± 0.1 [%]	P = 1016.7 ±	0.2 [mb]
$V_{\text{supply}} = 3.0 \text{ [V]}$	No. pulses = 20		
	me as sphere one's (see Ta	able 5a)	
•			V
t	V <sub>n</sub>	t	V <sub>n</sub>
33.73 ±0.01[mS]	00 ± 5 [mV]	34.03	-120
33.74	00	34.04	-120
33.75	- 20	34.05	-110
33.76	- 30	34.06	- 80
33.77	- 40	34.07	- 70
33.78	- 60	34.08	- 40
33.79	- 80	34.09	- 30
33.80	-100	34.10	- 20
33.81	-110	34.11	00
33.82	-120	34.12	00
33.83	-120	34.13	10
33.84	-110	34.14	00
33.85	-100	34.15	00
33.86	- 80	34.16	- 30
33.87	- 70	34.17	- 40
33.88	~ 50	34.18	- 70
33.89	- 40	34.19	- 90
33.90	- 10	34.20	-110
33.91	00	34.21	-120
33.92	00	34.22	-120
33.93	00	34.23	-120
33.94	00	34.24	-120
33.95	- 20	34.25	-110
33.96	- 30	34.26	- 80
33.97	- 50	34.27	- 70
33.98	- 70	34.28	- 40
33.99	- 80	34.29	- 30
34.00	-100	34.30	00
34.01	-110	34.31	00
34.02	-120	34.32	10

Table 10a: Noise Data for 3.5 [V] Supply Voltage; Collected 30 November 1987.

$T_c = 21.3 \pm 0.1 [^{\circ} c]$
$Rh = 51.3 \pm 0.1 [\%]$
$P = 1016.6 \pm 0.2 \text{ [mb]}$
No. pulses = 20
$V_{\text{supply}} = 3.5 \text{ [V]}$

t	$V_{\mathbf{t}}$	t	$V_{\mathbf{t}}$
2.24 ±0.01[mS]	740 ± 5 [mV]	2.44	750
2.25	830	2.45	830
2.26	820	2.46	810
2.27	720	2.47	700
2.28	530	2.48	500
2.29	380	2.49	360
2.30	180	2.50	160
2.31	- 60	2.51	- 80
2.32	-310	2.52	-330
2.33	-530	2.53	-540
2.34	-680	2.54	-690
2.35	-770	2.55	-760
2.36	-750	2.56	-740
2.37	-640	2.57	-630
2.38	-460	2.58	-460
2.39	-300	2.59	-280
2.40	-100	2.60	- 80
2.41	140	2.61	160
2.42	400	2.62	410
2.43	600	2.63	620

Table 10b: Noise Data for 3.5 [V] Supply Voltage Continued; Collected 30 November 1987.

t	V <sub>n</sub>	t	$V_n$
33.80 ±0.01[mS]	- 60 ± 5 [mV]	34.00	- 70
33.81	- 80	34.01	- 80
33.82	- 80	34.02	- 90
33.83	- 80	34.03	- 80
33.84	- 70	34.04	- 80
33.85	- 60	34.05	- 60
33.86	- 40	34.06	- 40
33.87	- 20	34.07	- 20
33.88	00	34.08	10
33.89	20	34.09	30
33.90	50	34.10	40
33.91	60	34.11	70
33.92	60	34.12	60
33.93	60	34.13	70
33.94	40	34.14	50
33.95	30	34.15	40
33.96	20	34.16	10
33.97	00	34.17	- 10
33.98	- 30	34.18	- 40
33.99	- 50	34.19	- 60

#### ONE WAY PROPAGATION PATH DATA

One way propagation path data are presented in the following tables.

 ${\rm T_c}$  and Rh are estimated. The estimates were based on hygrothermographic comparisons and 28 and 30 November data.

Table 11: One Way Propagation Path Data for 3.0 [V], 5000 [Hz] Continuous Wave Supply from the HP3314A Function Generator; Collected 7 December 1987.

```
(V_{rms})_t = 0.246 \pm 0.001 \text{ [V]}
T_c \approx 21.0 \text{ [° c]}
Rh \approx 50.0 \text{ [\%]}
P = 1021.5 \pm 0.2 \text{ [mb]}
R = 5.30 \text{ [m]} (calculated from two way propagation path data for spheres two, three and four)
```

i<sub>r</sub> (referenced to 10<sup>-12</sup> [W/m<sup>2</sup>]) 94.7 ± 0.1 [dB] 94.5 94.7 95.0 94.8 95.1

94.2

Table 12: One Way Propagation Path Data for 3.5 [V], 5000 [Hz] Continuous Wave Supply from the HP3314A Function Generator; Collected 7 December 1987.

```
(V_{rms})_t = 0.523 \pm 0.001 \text{ [V]}
T_c \approx 21.0 \text{ [° c]}
Rh \approx 50.0 \text{ [\%]}
P = 1021.5 \pm 0.2 \text{ [mb]}
R = 5.30 \text{ [m]} (calculated from two way propagation path data for spheres two, three and four)

I_r (referenced to 10^{-12} \text{ [W/m}^2\text{]})
97.4 \pm 0.1 \text{ [dB]}
96.4
```

95.1 97.0 96.2 97.1 96.4

95.4 96.3

97.4

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